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TABLES AND GRAPHS OF ZEROS OF CROSS PRODUCT BESSEL FUNCTIONS

$$J'_p(\xi)\,Y'_p(k\xi) - J'_p(k\xi)\,Y'_p(\xi) = 0$$

By

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TABLES AND GRAPHS OF ZEROS OF CROSS PRODUCT BESSEL FUNCTIONS $J'_p(\xi)\ Y'_p(k\xi) - J'_p(k\xi)\ Y'_p(\xi) = 0$

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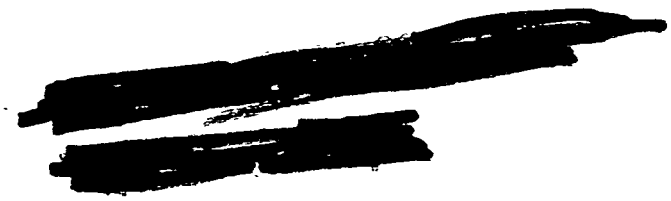
Helmut F. Bauer

ABSTRACT

15410

This paper represents the zeros and graphs of cross product Bessel functions as they occur in many applied problems. These roots are of interest in calculating frequencies for various modes of oscillations in a container comprised of a sector of annulus between two circular cylinders filled to a height, h, with incompressible liquid, and also in wave guide theory.



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AEROBALLISTICS DIVISION

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DEFINITION OF SYMBOLS

SYMBOL	DEFINITION
J_p, Y_p	Bessel functions of order p of the first and second kind
$k = \delta/a$	diameter ratio of inner - to outer tank
ξ	root of cross product Bessel functions $\Delta_p(\xi) = J'_p(\xi)\ Y'_p(k\xi) - J'_p(k\xi)\ Y'_p(\xi) = 0$
Γ	gamma function

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SUMMARY

Tables and graphs of zeros of the cross product Bessel function $J'_p(\xi)\ Y'_p(k\xi) - J'_p(k\xi)\ Y'_p(\xi) = 0$ are presented for a range in p from $1,(1),25$ and k from $0,(0.1),0.9$. These roots are of importance for many applied problems such as wave guide theory or in the calculation of frequencies for various modes of oscillations in a container comprised of a sector of annulus between two circular cylinders filled with incompressible liquid to a height, h.

SECTION I. INTRODUCTION

Interest has been expressed in the response of propellant in the containers of a space vehicle. With the increase of the tank diameters, the sloshing frequencies become lower and dangerously close to the control frequency. Subdivision of containers by longitudinal walls might be of importance to reduce the vibrating sloshing masses and increase the Eigen frequencies of the propellant, thus moving them farther away from the control frequency. In the determination of Eigen frequencies of the propellant in a circular cylindrical container of ring sector cross section filled to a height, h, with incompressible fluid the roots of cross product Bessel functions have to be known.

As vehicles lengthen, their fundamental bending frequencies become lower and the bending amplitudes of a space vehicle are of considerable magnitude. Therefore, circumstances of concern may arise from higher order modes.

SECTION II. RESULTS

In the calculation of the Eigen frequencies for various modes of oscillations in a circular cylindrical container of ring sector cross section filled to a height, h, with incompressible liquid, in wave guide theory, and in many other applied problems in mathematical physics involving annular or sectional cavities, the roots of the cross product of Bessel functions,

$$\Delta_p(\xi) = J'_p(k\xi)\ Y'_p(\xi) - J'_p(\xi)\ Y'_p(k\xi) = 0, \tag{1}$$

are of interest. The functions $J_p(\xi)$ and $Y_p(\xi)$ are Bessel functions of order p of the first and second kind, respectively. The value, k, is a parameter and $0 \leq k < 1$. In many cases where interest is expressed in the lower order mode, attention is directed primarily to the zeros of such cross products of Bessel functions when the subscript p is not large. There are, however, also physical and engineering problems, where circumstances of concern may arise from higher order modes.

In the case p = 0 the cross product of Bessel functions is

$$\Delta_0(\xi) = J_1(\xi)\ Y_1(k\xi) - J_1(k\xi)\ Y_1(\xi) = 0 \tag{2}$$

since $J'_0(\xi) = -J_1(\xi)$ and $Y'_0(\xi) = -Y_1(\xi)$.

For $k \to 0$ the roots of the cross product (1) approach the zeros of $J'_p(\xi) = 0$. This can be seen easily by rewriting equation (1) in the form

$$J'_p(\xi) = \frac{J'_p(k\xi)}{Y'_p(k\xi)} \cdot Y'_p(\xi).$$

For small values, k, the expression

$$\frac{J'_p(k\xi)}{Y'_p(k\xi)} = \frac{\pi x^p}{2^{2p}\,\Gamma(p)\,\Gamma(p+1)}$$

and approaches zero for $k \to 0$.

J. McMahon [1] gave an asymptotic expression for the roots of equation (1). However, the lowest root was not known, until D. O. North [2] and H. Buchholz [3] mentioned its existence. Curves showing values of the roots of the expression (1) are shown in a few cases by D. Kirkham [4] for p = 0, 1, 2, 3, 4. The purpose of this paper is to extend the range of the index p and present the ten lowest roots for p = 0, (1) to 25 for values of the parameter k between zero and unity. Bridge and Angrist [5] give eleven zeros for p = 0 (1) 12 and k = 1.1, 1.2, 1.5, (0.5) 5.0. Tables 1 through 13 extend the range of k and p. The calculation of the roots was accomplished by interpolation of the expression

$$\epsilon(\xi) = J'_p(\xi)\ Y'_p(k\xi) - J'_p(k\xi)\ Y'_p(\xi)$$

such that $\epsilon(\xi)$ approached zero. The parameter, k, changed from zero in steps of $\Delta k = 0.1$ to the value 0.9. The graphs (Fig. 1 through 26) represent the roots for various subscripts, p, as functions of the parameter, k. The circles that are indicated express computed points. In the case of p = 0 the lowest root disappears.

For the calculation of the derivatives of the Bessel functions, the series expansion was used in the argument range $0 \le x \le 5.0$ for the Bessel function of first kind and $0 \le x \le 5.75$ for the Bessel function of second kind. For larger arguments the asymptotic expansions were employed: [6]

$$J_p(x) = \left(\frac{2}{\pi x}\right)^{1/2} \left[\cos\left(x - \frac{p\pi}{2} - \frac{\pi}{4}\right) \alpha_p(x) - \sin\left(x - \frac{p\pi}{2} - \frac{\pi}{4}\right) \beta_p(x)\right]$$

$$Y_p(x) = \left(\frac{2}{\pi x}\right)^{1/2} \left[\cos\left(x - \frac{p\pi}{2} - \frac{\pi}{4}\right) \beta_p(x) + \sin\left(x - \frac{p\pi}{2} - \frac{\pi}{4}\right) \alpha_p(x)\right]$$

where

$$\alpha_p(x) = 1 + \sum_{\lambda=1}^{\infty} \frac{(-1)(4_p{}^2 - 1^2)(4_p{}^2 - 3^2)\cdots\cdots\cdots(4_p{}^2 - (4\lambda - 1)^2)}{(2\lambda)!\ 2^{6\lambda}\ x^{2\lambda}}$$

$$\beta_p(x) = -\sum_{\lambda=1}^{\infty} \frac{(-1)(4_p{}^2 - 1^2)(4_p{}^2 - 3^2)\cdots\cdots\cdots(4_p{}^2 - (4\lambda - 3)^2)}{(2\lambda - 1)!\ 2^{6\lambda - 3}\ x^{2\lambda - 1}}$$

For the cases in which argument and order of Bessel functions are nearly equal, the results of reference [6] have been used. In Tables 1 through 13 the lowest ten roots of equation (1) are given for $k = 0, 0.1, 0.2 \cdots 0.9$ and $p = 0, 1, 2, \cdots 25$. It should be noted that the roots are correct to at least the fourth digit.

Table 1

$\Delta_0(\ ) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	3.83171	3.94094	4.23575	4.70577	5.39118	6.39316	7.93008	10.52203	15.73755	31.42923
1	7.01559	7.33057	8.05536	9.10423	10.55773	12.62470	15.74728	20.96939	31.43083	62.83849
2	10.17347	10.74838	11.92658	13.55317	15.76646	18.88893	23.58833	31.43294	47.13383	94.25221
3	13.32369	14.18864	15.82104	18.01998	20.98820	25.16241	31.43576	41.90067	62.83931	125.66702
4	16.47063	17.64331	19.72706	22.49481	26.21548	31.43971	39.28579	52.37010	78.54578	157.08228
5	19.61586	21.10731	23.63949	26.97386	31.44561	37.71895	47.13713	62.84041	94.25276	188.49777
6	22.76008	24.57757	27.55584	31.45540	36.67739	43.99932	54.98923	73.31116	109.96001	219.91338
7	25.90367	28.05219	31.47472	35.93852	41.91022	50.28038	62.84179	83.78220	125.66744	251.32906
8	29.04683	31.52994	35.39534	40.42270	47.14374	56.56194	70.69471	94.25348	141.37499	282.74482
9	32.18968	35.01001	39.31719	44.90764	52.37775	62.84378	78.54780	104.72487	157.08262	314.16056

$\Delta_1(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	1.84118	1.80347	1.70512	1.58207	1.46178	1.35467	1.26208	1.18237	1.11337	1.05314
1	5.33144	5.13714	4.96086	5.13740	5.65910	6.56494	8.04109	10.59184	15.77771	31.44690
2	8.53632	8.19917	8.43307	9.30827	10.68325	12.70642	15.80106	21.00371	31.45076	62.84733
3	11.70601	11.35880	12.16505	13.68364	15.84808	18.94266	23.62392	31.45574	47.14711	94.25810
4	14.86359	14.63436	15.99323	18.11588	21.04879	25.20249	31.46238	41.91775	62.84927	125.67144
5	18.01553	17.98642	19.86163	22.57071	26.26370	31.47169	39.30706	52.38376	78.55375	157.08582
6	21.16437	21.38369	23.75000	27.03672	31.48568	37.74557	47.15485	62.85178	94.25939	188.50071
7	24.31133	24.80814	27.64965	31.50907	36.71167	44.02211	55.00440	73.32088	109.96569	219.91590
8	27.45705	28.24974	31.55626	35.98536	41.94018	50.30034	62.85507	83.79075	125.67241	251.33127
9	30.60192	31.70269	35.46747	40.46426	47.17035	56.57965	70.70650	94.26106	141.37941	282.74578

Table 2

$\Delta_2(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	3.05424	3.05294	3.03472	2.96850	2.84240	2.68120	2.51595	2.36285	2.22646	2.10622
1	6.70613	6.68669	6.49495	6.27367	6.41600	7.06258	8.36708	10.79885	15.89762	31.50001
2	9.96947	9.88752	9.54946	9.91800	11.05596	12.94941	15.96147	21.10636	31.51055	62.87589
3	13.17037	12.96963	12.89974	14.07466	16.09165	19.10316	23.73041	31.52410	47.18691	94.27578
4	16.34752	16.01187	16.51926	18.40313	21.22993	25.32243	31.54212	41.96895	62.87912	125.68471
5	19.51219	19.10106	20.26958	22.79807	26.40803	31.56748	39.37081	52.42470	78.57762	157.09642
6	22.67158	22.28166	24.08355	27.22504	31.60568	37.82532	47.20795	62.88587	94.27928	188.50955
7	25.82604	25.54689	27.93211	31.66989	36.81439	44.09043	55.04991	73.35016	109.98274	219.92350
8	28.97767	28.87353	31.80144	36.12574	42.02997	50.36011	62.89488	83.81631	125.68733	251.33790
9	32.12733	32.24178	35.68420	40.58883	47.25011	56.63273	70.74189	94.28380	141.39267	282.75268

$\Delta_3(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	4.20119	4.20115	4.19906	4.18011	4.10816	3.95776	3.75394	3.53961	3.33900	3.15933
1	8.01524	8.01416	7.96378	7.72126	7.53527	7.84011	8.88932	11.13634	16.09557	31.58836
2	11.34592	11.33836	11.10603	10.92017	11.66607	13.34760	16.22583	21.27642	31.60989	62.91809
3	14.58585	14.55603	14.14931	14.72716	16.49359	19.36843	23.90696	31.63759	47.25324	94.30525
4	17.78875	17.70576	17.43239	18.88109	21.52991	25.52135	31.67463	42.05416	62.92886	125.70680
5	20.97248	20.79509	20.96777	23.17609	26.64750	31.72662	39.47685	52.49288	78.61741	157.11411
6	24.14490	23.84611	24.64817	27.53818	31.80503	37.95793	47.29633	62.94269	94.31243	188.52428
7	27.31006	26.90842	28.40731	31.93739	36.98516	44.20410	55.12567	73.39883	110.01116	219.93612
8	30.47027	30.03007	32.21249	36.35930	42.17934	50.45953	62.96118	83.85893	125.71220	251.34894
9	33.62695	33.22727	36.04683	40.79615	47.38285	56.72114	70.80082	94.32167	141.41477	282.76250

Table 3

$\Delta_4(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	5.31756	5.31756	5.31735	5.31298	5.28210	5.17523	4.96965	4.71085	4.45074	4.21234
1	9.28240	9.28235	9.27339	9.15259	8.85259	8.83644	9.58236	11.59433	16.36888	31.71153
2	12.68191	12.68141	12.61018	12.24157	12.50097	13.89228	16.58998	21.51241	31.74843	62.97991
3	15.96411	15.96134	15.71267	15.64653	17.04913	19.73536	24.15224	31.79591	47.34583	94.34651
4	19.19603	19.18551	18.75480	19.55120	21.94613	25.79784	31.85931	42.17317	62.99841	125.73774
5	22.40103	22.37019	21.99309	23.70425	26.98064	31.94835	39.62487	52.58819	78.67308	157.13886
6	25.58976	25.51569	25.46419	27.97537	32.08282	38.14297	47.41981	63.02216	94.02216	188.54491
7	28.76784	28.61835	29.08547	32.31086	37.22339	44.36284	55.23158	73.46701	110.05094	219.95380
8	31.93854	31.68904	32.79478	36.68550	42.38788	50.59822	63.05392	83.91857	125.74700	251.36442
9	35.10392	34.74016	36.55836	41.08580	47.56828	56.84473	70.88322	94.37469	141.44571	282.77642

$\Delta_5(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	6.41565	6.41565	6.41563	6.41471	6.40311	6.24500	6.12301	5.86629	5.56194	5.26835
1	10.51932	10.51932	10.51796	10.21837	10.21837	9.98535	10.41790	12.22983	16.71450	31.86930
2	13.98706	13.98703	13.97125	13.69856	13.54274	14.57346	17.04980	21.81256	31.92576	63.05929
3	17.31279	17.31279	17.19262	16.82997	17.75361	20.20104	24.46464	31.99842	47.46467	94.39941
4	20.57549	20.57449	20.31383	20.41984	22.50000	26.15030	32.09544	42.32573	63.08768	125.77756
5	23.80357	23.79987	23.36430	24.38383	27.40612	32.23177	39.81450	52.71052	78.74460	157.17062
6	27.01030	26.99915	26.56888	28.57021	32.43810	38.37988	47.57817	63.12420	94.41846	188.57175
7	30.20284	30.11275	29.98912	32.79057	37.79057	44.56630	55.36749	73.55450	110.10204	219.97647
8	33.38544	33.28081	33.56011	37.10421	42.65510	50.77677	63.17293	83.99520	125.79172	251.38430
9	36.56077	36.41342	37.22527	41.45753	47.45753	57.00333	70.98907	94.44283	141.48540	282.79422

Table 4

$\Delta_6(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	7.50132	7.50134	7.50134	7.50115	7.49711	7.46206	7.26543	7.01548	6.66948	6.32054
1	11.73427	11.73427	11.73408	11.72070	11.54387	11.22684	11.37049	12.82714	17.12775	32.06113
2	15.26802	15.26802	15.26506	15.10890	14.75832	15.38161	17.59811	22.17418	32.14119	63.15620
3	18.63738	18.63737	18.61588	18.21929	18.60415	20.75996	24.84173	32.24427	47.60953	94.46429
4	21.93169	21.93161	21.80672	21.49687	23.12335	26.57612	32.38191	42.51147	63.19666	125.82619
5	25.18391	25.18355	24.92255	25.21973	27.92119	32.57546	40.04514	52.85966	78.83188	157.20956
6	28.40847	28.40847	27.97495	29.22246	32.86909	38.66782	47.77107	63.24871	94.49127	188.60390
7	31.61787	31.61389	31.15453	33.37547	37.89904	44.81392	55.53320	73.66134	110.16445	220.00455
8	34.81339	34.80280	34.53263	37.61442	42.98020	50.99393	63.31811	84.08875	125.84639	251.40809
9	37.99964	37.97469	38.06053	41.91051	48.09556	57.19666	71.11828	94.52604	141.53410	282.81542

$\Delta_7(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	8.52818	8.52818	8.52818	8.52815	8.53035	8.51207	8.34729	8.13979	7.77324	7.37455
1	12.93232	12.93232	12.93230	12.92963	12.50465	12.50810	12.41477	13.55920	17.60336	32.28621
2	16.52937	16.52937	16.53116	16.47634	16.08633	16.32141	18.32885	22.59421	32.39393	63.27052
3	19.94186	19.94186	19.93791	19.67414	19.60118	21.43978	25.28065	32.53247	47.78019	94.54062
4	23.26806	23.26805	23.24193	22.77787	23.85654	27.07221	32.71747	42.72997	63.32521	125.88342
5	26.54504	26.54501	26.41712	26.22487	28.52229	32.97775	40.31614	53.03530	78.93494	157.25541
6	29.79075	29.79063	29.53213	30.02628	33.37369	39.00576	47.99813	63.39553	94.57728	188.64193
7	33.01518	33.01685	32.58714	34.06408	38.33391	45.10509	55.72844	73.78741	110.23828	220.03730
8	36.22438	36.22446	35.74652	38.21470	43.36222	51.24957	63.48930	84.19917	125.91093	251.43713
9	39.42228	39.41918	39.09105	42.44366	48.43610	57.42443	71.27068	94.62426	141.59147	282.84119

Table 5

$\Delta_8(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	9.59581	9.59581	9.59581	9.59581	9.59543	9.58985	9.52458	9.26081	8.87557	8.42639
1	14.11541	14.11451	14.11541	14.11729	14.07608	13.78943	13.52760	14.37244	18.13837	32.54412
2	17.77401	17.77401	17.77393	17.75576	17.44960	17.34098	18.93491	23.07056	32.68324	63.40220
3	21.22907	21.22907	21.23067	21.07632	20.73118	22.13215	25.77957	32.86214	47.97633	94.62877
4	24.58720	24.58720	24.58173	24.19986	24.78305	27.63720	33.10106	42.98080	63.47324	125.94973
5	27.88928	27.88927	27.85966	27.40058	29.22874	33.43755	40.62688	53.23736	79.05370	157.30860
6	31.15533	31.15532	31.02580	30.99977	33.95150	39.39298	48.25889	63.56458	94.67636	188.68666
7	34.39663	34.39659	34.14235	34.85442	38.83241	45.43924	55.95296	73.93263	110.32328	220.07504
8	37.62008	37.61992	37.20018	38.92314	43.80062	51.54330	63.68634	84.32645	125.98544	251.47070
9	40.83018	40.83235	40.34370	43.05818	48.82728	57.68636	71.44615	94.73748	141.65768	282.87043

$\Delta_9(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	10.65808	10.65808	10.65808	10.65808	10.65796	10.65848	10.61280	10.36868	9.97542	9.47824
1	15.28660	15.28659	15.28659	15.26834	15.27005	15.04807	14.68891	15.34073	18.72746	32.83402
2	19.00458	19.00458	19.00457	19.00004	18.78937	18.46671	19.70005	23.59981	33.00815	63.55112
3	22.50140	22.50140	22.50124	22.44752	21.98444	22.93872	26.33540	33.23205	48.19770	94.72863
4	25.89128	25.89128	25.89263	25.64732	25.69500	28.26800	33.53115	43.26341	63.64061	126.02472
5	29.21857	29.21857	29.21174	28.74320	29.97701	33.95296	40.97652	53.46548	79.18808	157.36862
6	32.50525	32.50525	32.47302	32.08337	34.60067	39.82825	48.55295	63.75560	94.78857	188.73626
7	35.76379	35.76379	35.63368	35.81152	39.39309	45.81559	56.20644	74.09686	110.41956	220.11816
8	39.00190	39.00189	38.75294	39.68879	44.29431	51.87455	63.90897	84.47046	126.06977	251.50790
9	42.22464	42.22458	41.81395	43.75401	49.26827	57.98204	79.76114	94.86566	141.73268	282.90414

Table 6

$\Delta_{10}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	11.71594	11.71595	11.71595	11.71594	11.71591	11.71423	11.68836	11.46351	11.07246	10.53004
1	16.44767	16.44767	16.44767	16.44761	16.44246	16.27354	15.88091	16.23428	19.36610	33.15495
2	20.22302	20.22301	20.22301	20.22451	20.07983	19.66846	20.53696	24.17892	33.36766	63.71714
3	23.76072	23.76072	23.76069	23.74075	23.31729	23.90103	26.94536	33.64098	48.44391	94.84003
4	27.18202	27.18202	27.18175	27.03833	26.75734	28.96216	34.00621	43.57721	63.82712	126.10838
5	30.53451	30.53451	30.53559	30.18108	30.82875	34.52225	41.36418	53.71935	79.33800	157.43561
6	33.84197	33.84197	33.83400	33.33481	35.32011	40.31042	48.87972	63.96850	94.91384	188.79240
7	37.11800	37.11800	37.08387	36.81437	40.01486	46.23334	56.48853	74.28004	110.52711	220.16617
8	40.37107	40.37107	40.24117	40.56732	44.84234	52.24278	64.15696	84.63109	126.16393	251.55002
9	43.60677	43.60676	43.36375	44.52941	49.75831	58.31107	71.86576	95.00873	141.81650	282.94103

$\Delta_{11}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	12.77009	12.77010	12.77010	12.77010	12.77009	12.76932	12.75478	12.60944	12.16639	11.58176
1	17.60005	17.60006	17.60006	17.60004	17.60100	17.50975	17.08930	17.20210	20.05001	33.50622
2	21.43083	21.43083	21.43083	21.43031	21.37283	20.92092	21.51504	24.80488	33.76070	63.90007
3	25.00852	25.00852	25.00851	25.00315	24.67699	24.82253	27.60674	34.08765	48.71470	94.96297
4	28.46086	28.46086	28.46081	28.40938	27.93805	29.73125	34.52471	43.92159	64.03262	126.20085
5	31.83843	31.83843	31.83802	31.61789	31.81489	35.14373	41.78896	53.99861	79.50338	157.50974
6	35.16672	35.16672	35.16754	34.71775	36.10525	40.83828	49.23863	64.20297	95.05207	188.85406
7	38.46039	38.46039	38.45148	37.98094	40.69675	46.69165	56.79889	74.48199	110.64585	220.21983
8	41.72863	41.72863	41.69318	41.58437	45.44381	52.64738	64.43008	84.80834	126.26793	251.59612
9	44.97753	44.97753	44.87616	45.38624	50.29662	58.67300	72.10949	95.16663	141.90900	282.98275

Table 7

$\Delta_{12}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	13.82109	13.82110	13.82110	13.82110	13.82110	13.82074	13.81458	13.69506	13.25695	12.63334
1	18.74485	18.74485	18.74485	18.74485	18.74373	18.69147	18.30256	18.21969	20.77515	33.88688
2	22.62927	22.62927	22.62927	22.62913	22.60156	22.19851	22.43537	25.47472	34.18610	64.09988
3	26.24604	26.24604	26.24604	26.24364	26.01879	25.84935	28.31696	34.57062	49.00954	95.09750
4	29.72898	29.72898	29.72897	29.70906	29.21272	30.52270	35.08504	44.29590	64.25705	126.30218
5	33.13145	33.13145	33.13137	33.02564	32.82150	35.81573	42.24992	54.30292	79.68414	157.59069
6	36.48055	36.48055	36.48001	36.16124	36.95655	41.41058	49.62907	64.45890	95.20328	188.92164
7	39.79194	39.79194	39.78922	39.28768	41.43807	47.18961	57.13701	74.70257	110.77575	220.27681
8	43.07549	43.07549	43.06583	42.67523	46.09788	53.08769	64.72800	85.00206	126.38176	251.64686
9	46.33777	46.33777	46.30148	46.38730	50.88245	59.06736	72.37558	95.33922	142.01032	283.02765

$\Delta_{13}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	14.86936	14.86936	14.86936	14.86936	14.86936	14.86920	14.86271	14.77171	14.34391	13.68478
1	19.88295	19.88296	19.88296	19.88296	19.88253	19.85297	19.51179	19.27319	21.53781	34.29580
2	23.81936	23.81936	23.81936	23.81932	23.80794	23.47827	23.43437	26.18569	34.64280	64.31640
3	27.47433	27.47433	27.47433	27.47357	27.34647	26.95685	29.07359	35.08862	49.32803	95.24350
4	30.98739	30.98739	30.98739	30.98197	30.54893	31.38885	35.68561	44.69932	64.50003	126.41211
5	34.41455	34.41455	34.41453	34.36698	33.94066	36.53678	42.74592	54.63189	79.88012	157.67902
6	37.78438	37.78438	37.78426	37.58562	37.87860	42.02614	50.05040	64.73593	95.36732	188.99522
7	41.11351	41.11351	41.11283	40.69699	42.24328	47.72633	57.50253	74.94160	110.91677	220.33989
8	44.41245	44.41246	44.40935	43.90427	46.80387	53.56306	65.05047	85.21215	126.50542	251.70260
9	47.68825	47.68826	47.67802	47.40937	51.51509	59.49364	72.66383	95.52653	142.12031	283.07687

Table 8

$\Delta_{14}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	15.91522	15.91523	15.91523	15.91523	15.91523	15.91516	15.91120	15.84060	15.42709	14.73604
1	21.01512	21.01512	21.01512	21.01512	21.01495	20.99983	20.71056	20.35415	22.33493	34.73213
2	25.00193	25.00194	25.00194	25.00193	24.99599	24.74335	24.49131	26.93516	35.12956	64.54941
3	28.69426	28.69426	28.69426	28.69403	28.62366	28.12904	29.87785	35.64021	49.66981	95.40089
4	32.23697	32.23697	32.23697	32.23420	31.90530	32.30956	36.32484	45.13126	64.76148	126.53073
5	35.68855	35.68855	35.68854	35.66971	35.15348	37.30554	43.27607	54.98505	80.09128	157.77412
6	39.07900	39.07900	39.07897	38.98498	38.91450	42.68372	50.50192	65.03390	95.54423	189.07454
7	42.42585	42.42586	42.42570	42 13919	43.08744	48.30089	57.89499	75.19898	111.06888	220.40819
8	45.74024	45.74024	45.73942	45.25271	47.56133	54.07278	65.39707	85.43849	126.63875	251.76233
9	49.02963	49.02964	49.02618	48.56718	52.19392	59.95132	72.97398	95.72842	142.23900	283.12968

$\Delta_{15}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	16.95879	16.95898	16.95898	16.95898	16.95898	16.95895	16.95655	16.90293	16.50633	15.78709
1	22.14194	22.14194	22.14194	22.14194	22.14188	22.13612	21.89494	21.45654	23.13961	35.19484
2	26.17774	26.17726	26.17773	26.17772	26.17512	25.99997	25.59369	27.72067	35.64523	64.79874
3	29.90658	29.90658	29.90658	29.90651	29.87024	29.35227	30.70323	36.22404	50.03438	95.56972
4	33.47845	33.47845	33.47845	33.47750	33.24595	33.33584	37.00122	45.59088	65.04115	26.65807
5	36.95417	36.95417	36.95417	36.94922	36.44269	38.12087	43.83931	55.36195	80.31747	157.87620
6	40.36510	40.36510	40.36510	40.32206	39.98123	43.38224	50.98296	65.35249	95.73384	189.15973
7	43.72963	43.72936	43.72960	43.55047	44.00100	48.91236	58.31388	75.47466	111.23197	220.48125
8	47.05946	47.05946	47.05926	46.67801	48.37012	54.61615	65.76755	85.68093	126.78188	251.82600
9	50.36251	50.36252	50.36157	49.84591	52.91841	60.43982	73.30579	95.94484	142.36649	283.18672

Table 9

$\Delta_{16}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	18.10419	18.10419	18.10419	18.10419	18.10419	18.10418	17.99938	17.95978	17.58152	16.83793
1	23.26394	23.26394	23.26394	23.26394	23.26392	23.25880	23.09051	22.57556	23.98660	35.68295
2	27.34733	27.34734	27.34734	27.34734	27.34623	27.23286	26.73261	28.53740	36.18861	65.06423
3	31.11194	31.11194	31.11194	31.11192	31.09519	30.61061	31.58021	36.83872	50.42121	95.74980
4	34.71248	34.71248	34.71248	34.71217	34.57596	34.36181	37.71328	46.07746	65.33884	126.79394
5	38.21206	38.21206	38.21206	38.20911	37.78122	38.96933	44.43459	55.76222	80.55865	157.98519
6	41.64331	41.64331	41.64331	41.62614	41.13832	44.12067	51.49280	65.69148	95.93618	189.25074
7	45.02543	45.02543	45.02542	44.94252	44.99998	49.55986	58.75873	75.76792	111.40610	220.55922
8	48.37069	48.37069	48.37065	48.11428	49.22194	55.19245	66.16153	85.93942	126.93462	251.89423
9	51.68742	51.68742	51.68718	51.22555	53.68827	60.95863	73.65900	96.17563	142.50252	283.24763

$\Delta_{17}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	19.14730	19.14731	19.14731	19.14731	19.14731	17.14730	19.14626	19.11250	18.68955	17.88853
1	24.38156	24.38156	24.38156	24.38156	24.38155	24.37882	24.25285	23.70712	24.86047	36.19540
2	28.51132	28.51131	28.51131	28.51131	28.51085	28.43816	27.90044	29.37159	36.75854	65.34564
3	32.31088	32.31088	32.31088	32.31088	32.30186	31.88636	32.49454	37.48292	50.82989	95.94112
4	35.93963	35.93963	35.93963	35.93953	35.86060	35.45506	38.49566	46.59019	65.65425	126.93840
5	39.46277	39.46277	39.46277	39.46170	39.13455	39.86928	45.06088	56.18536	80.81444	158.10125
6	42.91415	42.91415	42.91415	42.90650	42.37400	44.89812	52.03075	66.05045	96.15107	189.34767
7	46.31377	46.31377	46.31377	46.27541	46.05144	50.24253	59.22902	76.07909	111.59117	220.64243
8	49.67443	49.67443	49.67442	49.53322	50.12732	55.80098	66.57863	86.21372	127.09706	251.96696
9	53.00485	53.00484	53.00479	52.65882	54.50342	61.50717	74.03333	96.42074	142.64722	283.31206

Table 10

$\Delta_{18}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	20.18882	20.18882	20.18882	20.18882	20.18882	20.18881	20.18819	20.16522	19.76178	18.93887
1	25.49518	25.49519	25.49519	25.49519	25.49519	25.49374	25.40192	24.84766	25.75599	36.73124
2	29.67010	29.67010	29.67010	29.67010	29.66990	29.62530	29.08954	30.24550	37.35388	65.64281
3	33.50392	33.50392	33.50392	33.50392	33.49967	33.16288	33.44282	38.15534	51.25981	96.14369
4	37.16041	37.16040	37.16040	37.16037	37.11760	36.60462	39.23910	47.12830	65.98715	127.09156
5	40.70680	40.70680	40.70680	40.70643	40.47296	40.81193	45.71717	56.63088	81.08497	158.22427
6	44.17813	44.17813	44.17813	44.17515	43.67389	45.71387	52.59600	66.42921	96.37847	189.45044
7	47.59513	47.59513	47.59513	47.57995	47.15989	50.95955	59.72423	76.40784	111.78717	220.73046
8	50.97113	50.97113	50.97113	50.89845	51.08532	56.44105	67.01848	86.50378	127.26913	252.04426
9	54.31522	54.31522	54.31520	54.08617	55.36410	62.08488	74.42852	96.68007	142.80052	283.38059

$\Delta_{19}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	21.22887	21.22887	21.22887	21.22887	21.22887	21.22887	21.22849	21.20697	20.83069	19.98893
1	26.60515	26.60515	26.60515	26.60515	26.60515	26.60439	26.53913	25.99403	26.67055	37.28943
2	30.82410	30.82410	30.82410	30.82410	30.82402	30.79830	30.29232	31.14710	37.97348	65.95552
3	34.69147	34.69147	34.69147	34.69147	34.68951	34.42744	34.48825	38.85476	51.71052	96.35732
4	38.37523	38.37523	38.37523	38.37522	38.35416	37.80222	40.05045	47.69100	66.33731	127.25323
5	41.94459	41.94459	41.94459	41.94446	41.80349	41.85536	46.40250	57.09829	81.37001	158.35415
6	45.43566	45.43567	45.43567	45.43455	45.01396	46.56736	53.18792	66.82742	96.61831	189.55893
7	48.86993	48.86993	48.86993	48.86268	48.34483	51.71019	60.24384	76.75389	111.99396	220.82364
8	52.26121	52.26121	52.26121	52.22744	52.14626	57.11194	67.48070	86.80940	127.45073	252.12592
9	55.61893	55.61893	55.61893	55.49570	56.25661	62.69118	74.84425	96.95352	142.96241	283.45330

Table 11

$\Delta_{20}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	22.26759	22.26759	22.26759	22.26759	22.26759	22.26759	22.26736	22.25125	21.89625	21.03870
1	27.71172	27.71172	27.71172	27.71172	27.71172	27.71133	27.66629	27.14352	27.60219	37.86904
2	31.97366	31.97366	31.97366	31.97366	31.97363	31.95704	31.50159	32.07265	38.61625	66.28352
3	35.87393	35.87393	35.87393	35.87393	35.87304	35.69278	35.50631	39.57993	52.18154	96.58201
4	39.58453	39.58453	39.58453	39.58452	39.57272	39.03822	40.89271	48.27750	66.70446	127.42350
5	43.17654	43.17654	43.17654	43.17650	43.09199	42.87806	47.11591	57.58712	81.66935	158.49082
6	46.68717	46.68717	46.68717	46.68676	46.36445	47.44313	53.80573	67.24467	96.87046	189.67308
7	50.13856	50.13856	50.13856	50.13564	49.59792	52.49378	60.78733	77.11706	112.21154	220.92170
8	53.54503	53.54503	53.54503	53.53199	53.21109	57.81305	67.96490	87.13045	127.64200	252.21170
9	56.91635	56.91635	56.91635	56.85295	57.20407	63.32551	75.28019	97.24092	143.13286	283.52965

$\Delta_{21}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	23.30501	23.30508	23.30508	23.30508	23.30508	23.30508	23.30495	23.29292	22.95855	22.08816
1	28.81517	28.81517	28.81517	28.81517	28.81517	28.81497	28.78518	28.29375	28.66122	38.46916
2	33.11910	33.11911	33.11911	33.11910	33.11909	33.10944	32.71102	33.02025	39.28113	66.62663
3	37.05164	37.05163	37.05163	37.05163	37.05124	36.92540	36.56579	40.32975	52.67223	96.81769
4	40.78865	40.78864	40.78864	40.78864	40.78278	40.30024	41.76486	48.88704	67.08833	127.60215
5	44.40300	44.40300	44.40300	44.40299	44.35549	43.95948	47.85651	58.09680	81.98293	158.63447
6	47.93298	47.93298	47.93298	47.93283	47.70023	48.36489	54.44873	67.68075	97.13490	189.79318
7	51.40137	51.40137	51.40137	51.40023	50.90597	53.30980	61.35411	77.49713	112.43981	221.02468
8	54.82293	54.82293	54.82293	54.81618	54.35069	58.54370	68.47061	87.46672	127.84263	252.30213
9	58.20780	58.20780	58.20780	58.17843	58.19621	63.98732	75.73612	97.54219	143.31184	283.61018

Table 12

$\Delta_{22}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	24.34144	24.34144	24.34144	24.34144	24.34144	24.34144	24.34136	24.33241	24.11317	23.13729
1	29.91571	29.91571	29.91571	29.91571	29.91571	29.91561	29.89314	29.44269	29.61893	39.08881
2	34.26071	34.26071	34.26071	34.26071	34.26070	34.25515	33.91543	34.07112	39.96710	66.98462
3	38.22488	38.22488	38.22488	38.22488	38.22471	38.14047	37.65999	41.10316	53.18213	97.06423
4	41.98788	41.98788	41.98788	41.98788	41.98503	41.57441	42.64435	49.51885	67.48861	127.78928
5	45.62431	45.62431	45.62431	45.62431	45.59981	45.09104	48.62346	58.62691	82.31053	158.78505
6	49.17342	49.17342	49.17342	49.17337	49.03039	49.32202	55.11623	68.13522	97.41142	189.91900
7	52.65866	52.65866	52.65866	52.65823	52.24700	54.15786	61.94368	77.89381	112.67872	221.13273
8	56.09523	56.09524	56.09524	56.09244	55.55737	59.30339	68.99753	87.81810	128.05282	252.39662
9	59.49359	59.49359	59.49359	59.47937	59.28707	64.67609	76.21167	97.85723	143.49931	283.69415

$\Delta_{23}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	25.37676	25.37676	25.37676	25.37676	25.37676	25.37676	25.37671	25.37007	25.17347	24.18608
1	31.01355	31.01355	31.01355	31.01355	31.01355	31.01350	30.99821	30.58867	30.59921	39.72708
2	35.39872	35.39872	35.39872	35.39872	35.39872	35.39555	35.11102	35.06172	40.67319	67.35719
3	39.39396	39.39396	39.39396	39.39396	39.39389	39.33968	38.78415	41.89914	53.71072	97.32157
4	43.18254	43.18254	43.18254	43.18254	43.18118	42.84770	43.56799	50.17218	67.90054	127.98475
5	46.84076	46.84075	46.84075	46.84075	46.82652	46.26694	49.41598	59.17689	82.65200	158.94237
6	50.40880	50.40880	50.40880	50.40878	50.32083	50.31234	55.80756	68.60786	97.70001	190.05076
7	53.91074	53.91074	53.91074	53.91058	53.59477	55.03774	62.55551	78.30694	112.92825	221.24584
8	57.36222	57.36222	57.36222	57.36110	56.82427	60.09159	69.54514	88.18441	128.27239	252.49567
9	60.77400	60.77399	60.77399	60.76779	60.38580	65.39130	76.70654	98.18587	143.69530	283.78239

Table 13

$\Delta_{24}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	26.41110	26.41110	26.41110	26.41110	26.41110	26.41110	26.41108	26.40615	26.23083	25.23451
1	32.10885	32.10886	32.10886	32.10885	32.10886	32.10883	32.09859	31.73031	31.59734	40.38312
2	36.53336	36.53336	36.53336	36.53336	36.53336	36.53158	36.31629	36.08033	41.39838	67.74421
3	40.55913	40.55912	40.55912	40.55912	40.55909	40.52595	39.93415	42.71678	54.25746	97.58965
4	44.37289	44.37289	44.37289	44.37289	44.37225	44.11009	44.51707	50.84630	68.33732	128.18871
5	48.05260	48.05260	48.05260	48.05260	48.04526	47.48123	50.23337	59.74624	83.00719	159.10660
6	51.63937	51.63937	51.63937	51.63936	51.58850	51.39770	56.52203	69.09816	98.00050	190.18795
7	55.15787	55.15787	55.15787	55.15781	54.94525	55.93246	63.18908	78.73622	113.18827	221.36387
8	58.62415	58.62415	58.62415	58.62371	58.13871	60.90789	70.11309	88.56544	128.50134	252.59904
9	62.04927	62.04927	62.04927	62.04665	61.54997	66.13249	77.22040	98.52801	143.89967	283.87439

$\Delta_{25}(\xi) = 0$

n \ k	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	27.44454	27.44454	27.44454	27.44454	27.44454	27.44454	27.44453	27.44088	27.28537	26.28257
1	33.20179	33.20179	33.20179	33.20179	33.20179	33.20178	33.19492	32.86658	32.61001	41.05620
2	37.66484	37.66484	37.66484	37.66484	37.66484	37.66384	37.49374	37.12203	42.14189	68.14532
3	41.72057	41.72057	41.72057	41.72057	41.72056	41.69828	41.10564	43.55520	54.82184	97.86829
4	45.55917	45.55916	45.55916	45.55916	45.55887	45.37585	45.49000	51.54053	68.78522	128.40096
5	49.26009	49.26009	49.26009	49.26008	49.25636	48.72579	51.07498	60.33450	83.37593	159.27768
6	52.86538	52.86538	52.86538	52.86538	52.83825	52.46887	57.25904	69.60590	98.31287	190.33110
7	56.40029	56.40029	56.40029	56.40027	56.25711	56.86928	63.84383	79.18144	113.45870	221.48683
8	59.88125	59.88126	59.88126	59.88110	59.48020	61.75197	70.70100	88.96104	128.73959	252.70701
9	63.31966	63.31966	63.31966	63.31858	62.77430	66.89916	77.75292	98.88350	144.11241	283.97034

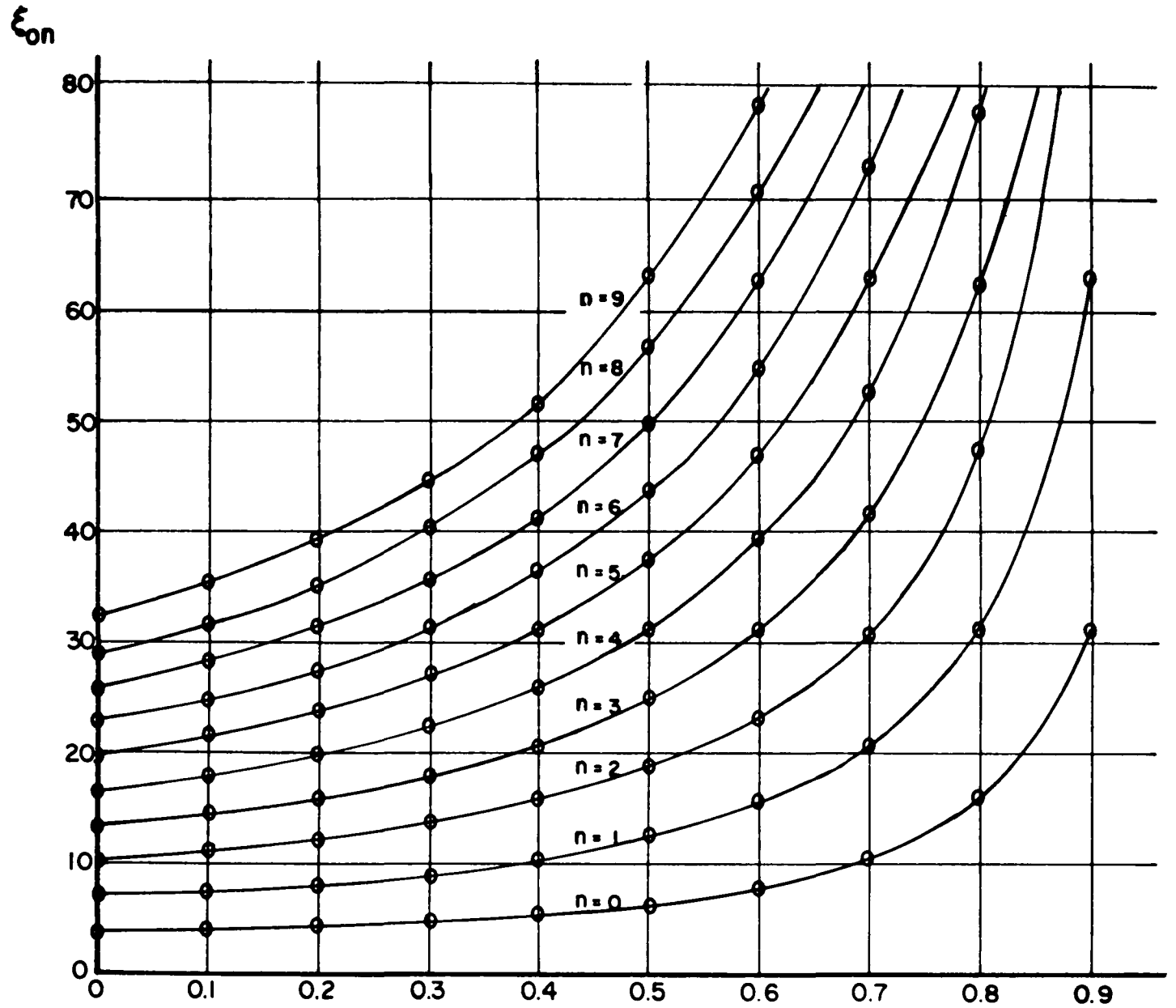

FIGURE 1. ROOTS $\Delta_0(\xi) = 0$ VS PARAMETER k

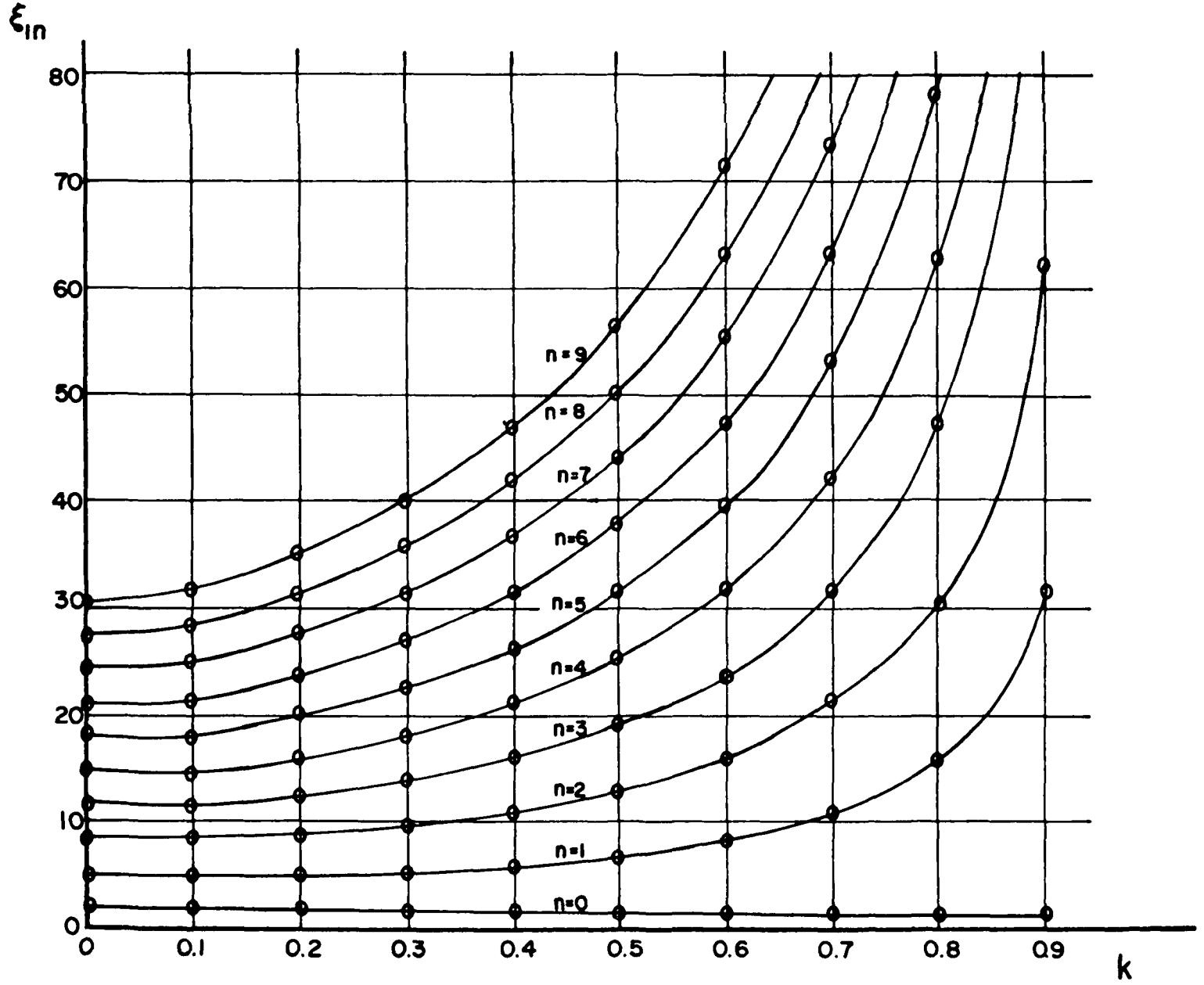

FIGURE 2. ROOTS $\Delta_1(\xi) = 0$ VS PARAMETER k

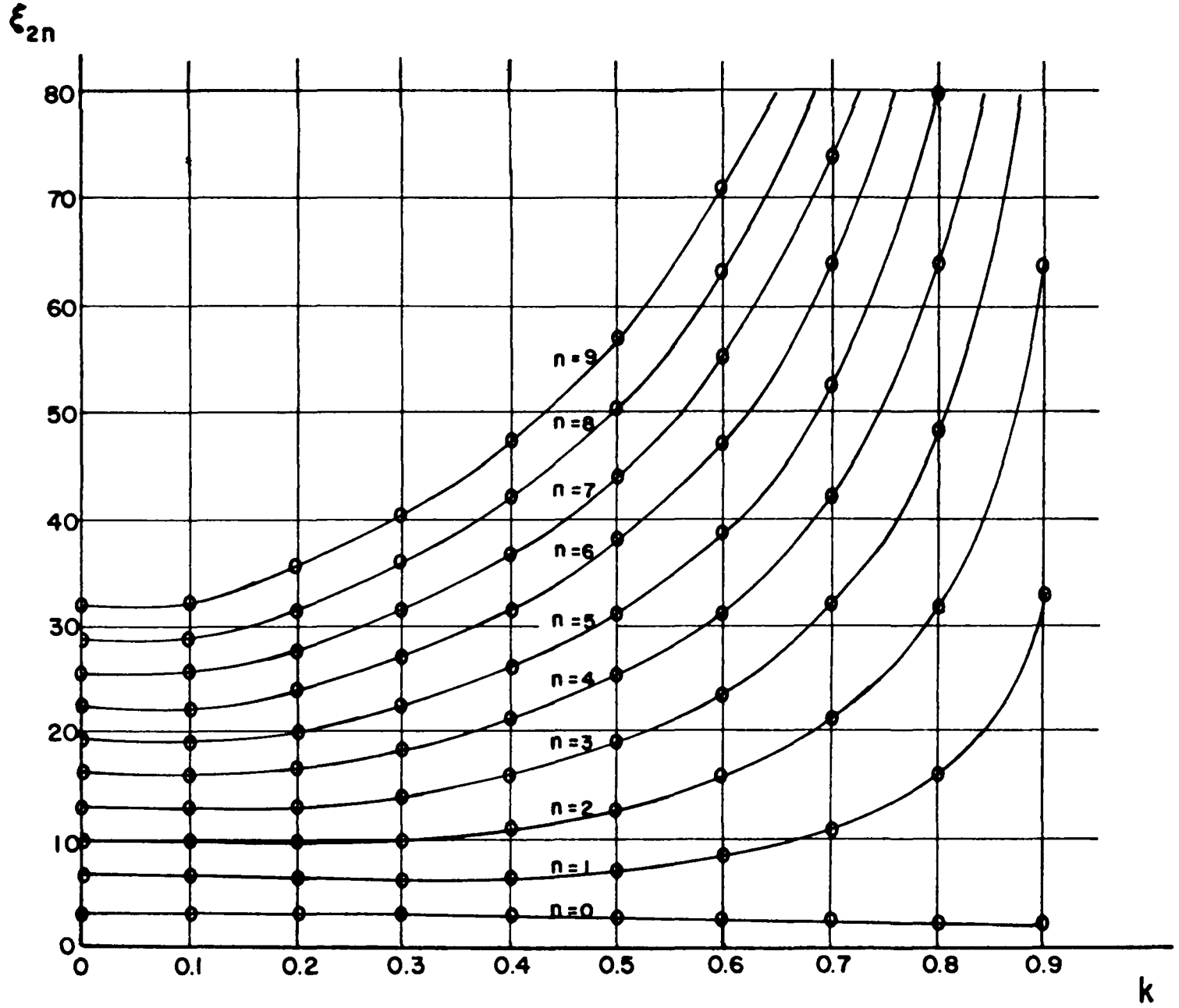

FIGURE 3. ROOTS $\Delta_2(\xi) = 0$ VS PARAMETER k

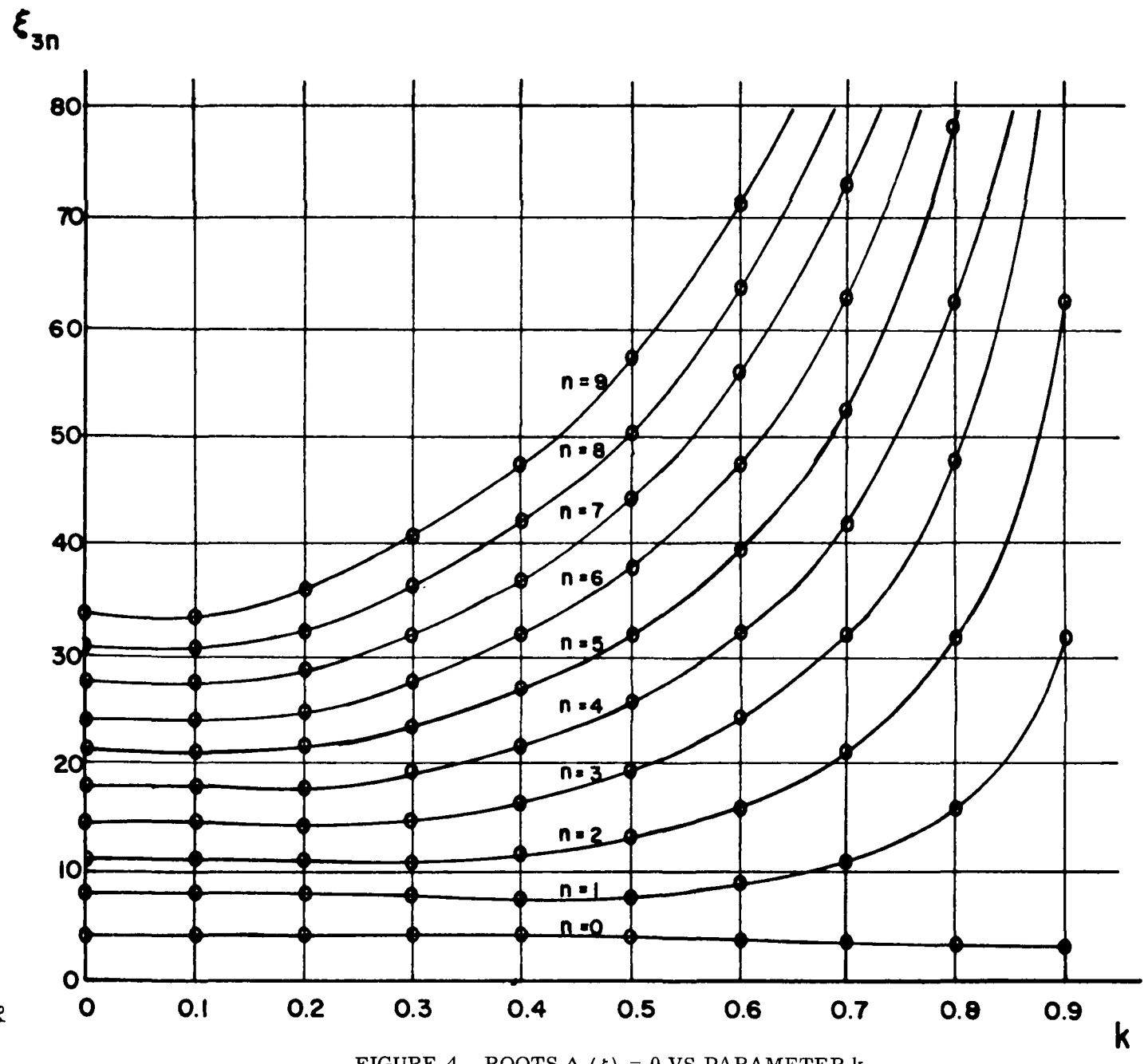

FIGURE 4. ROOTS $\Delta_3(\xi) = 0$ VS PARAMETER k

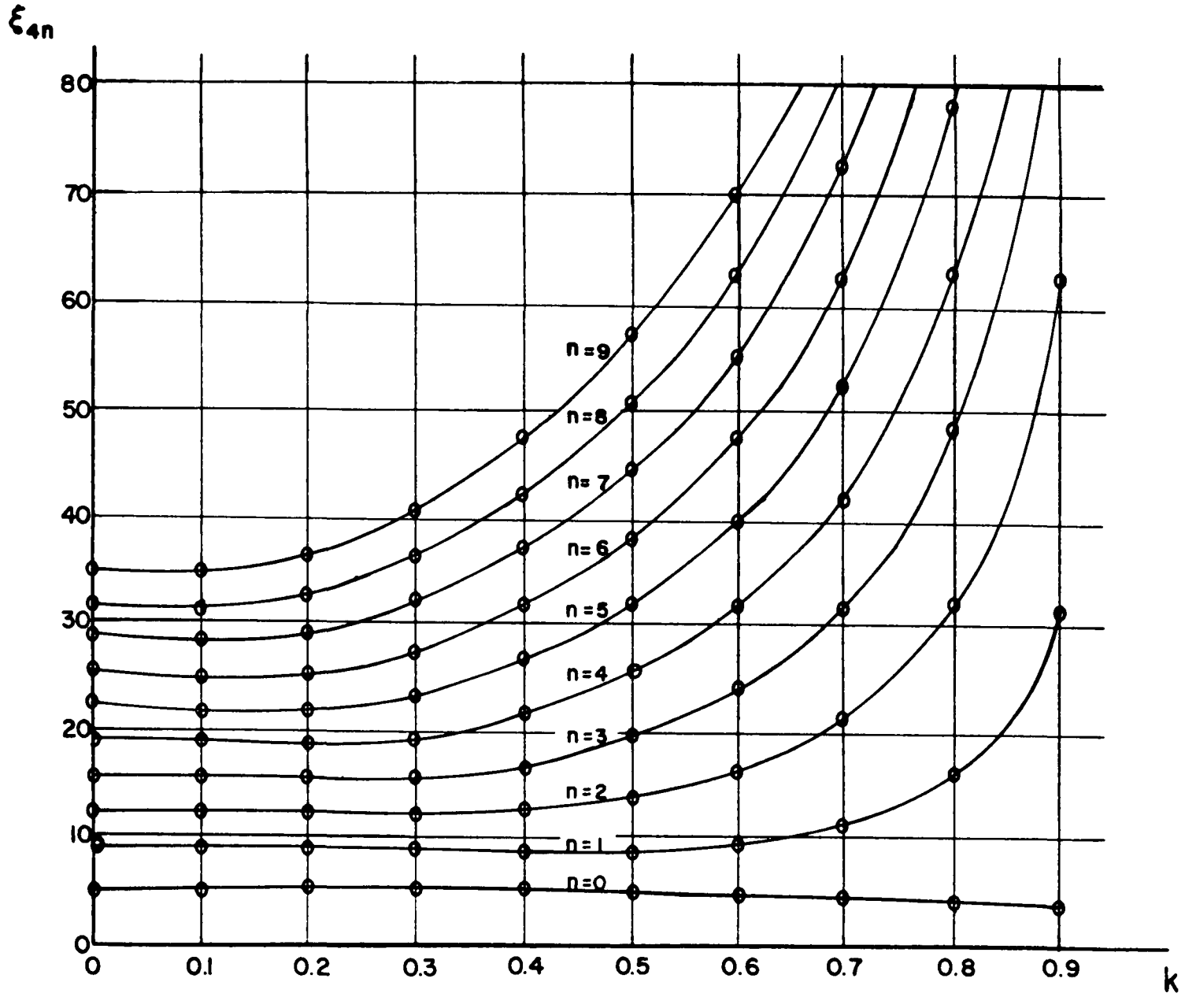

FIGURE 5. ROOTS $\Delta_4(\xi) = 0$ VS PARAMETER k

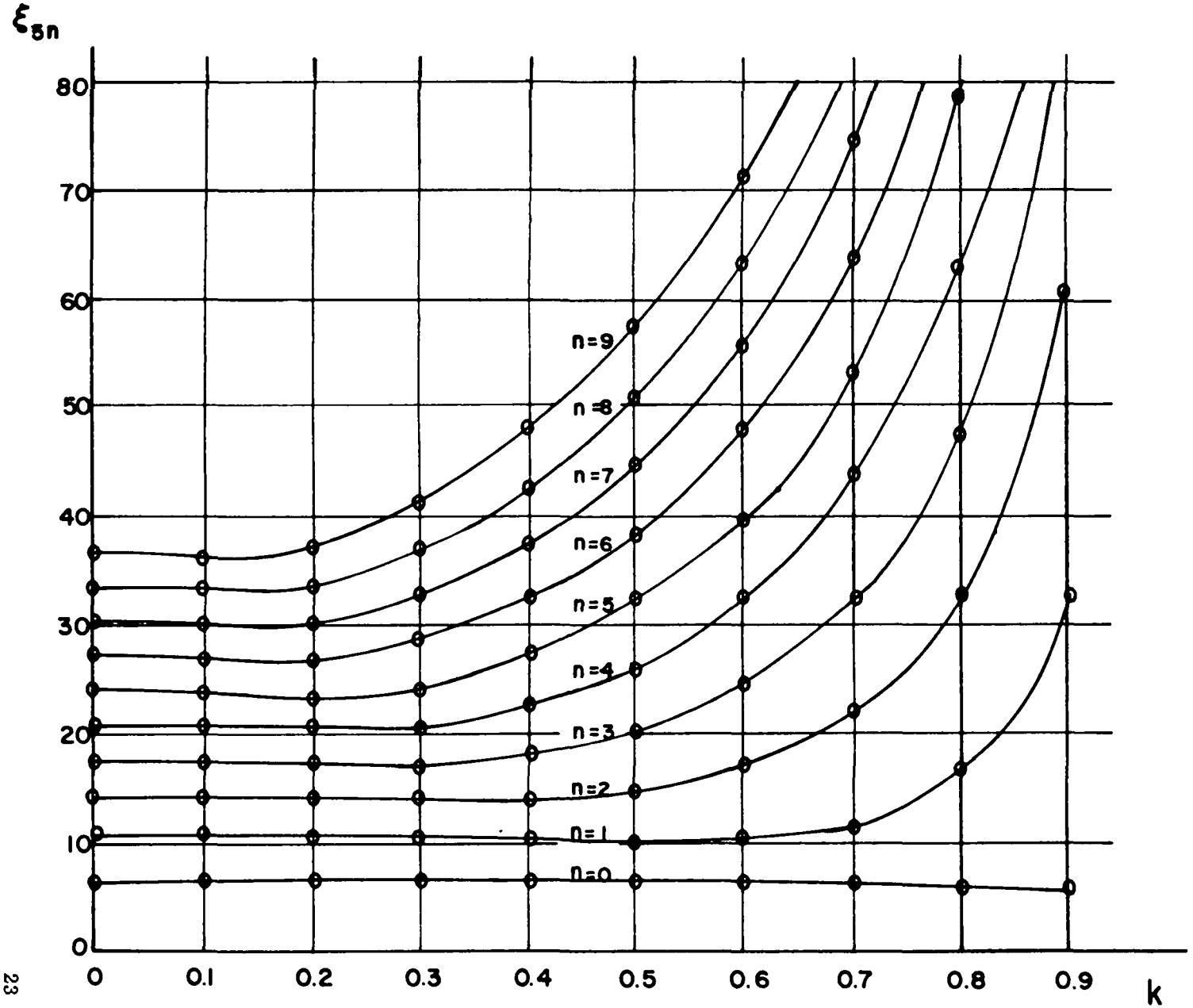

FIGURE 6. ROOTS $\Delta_5(\xi) = 0$ VS PARAMETER k

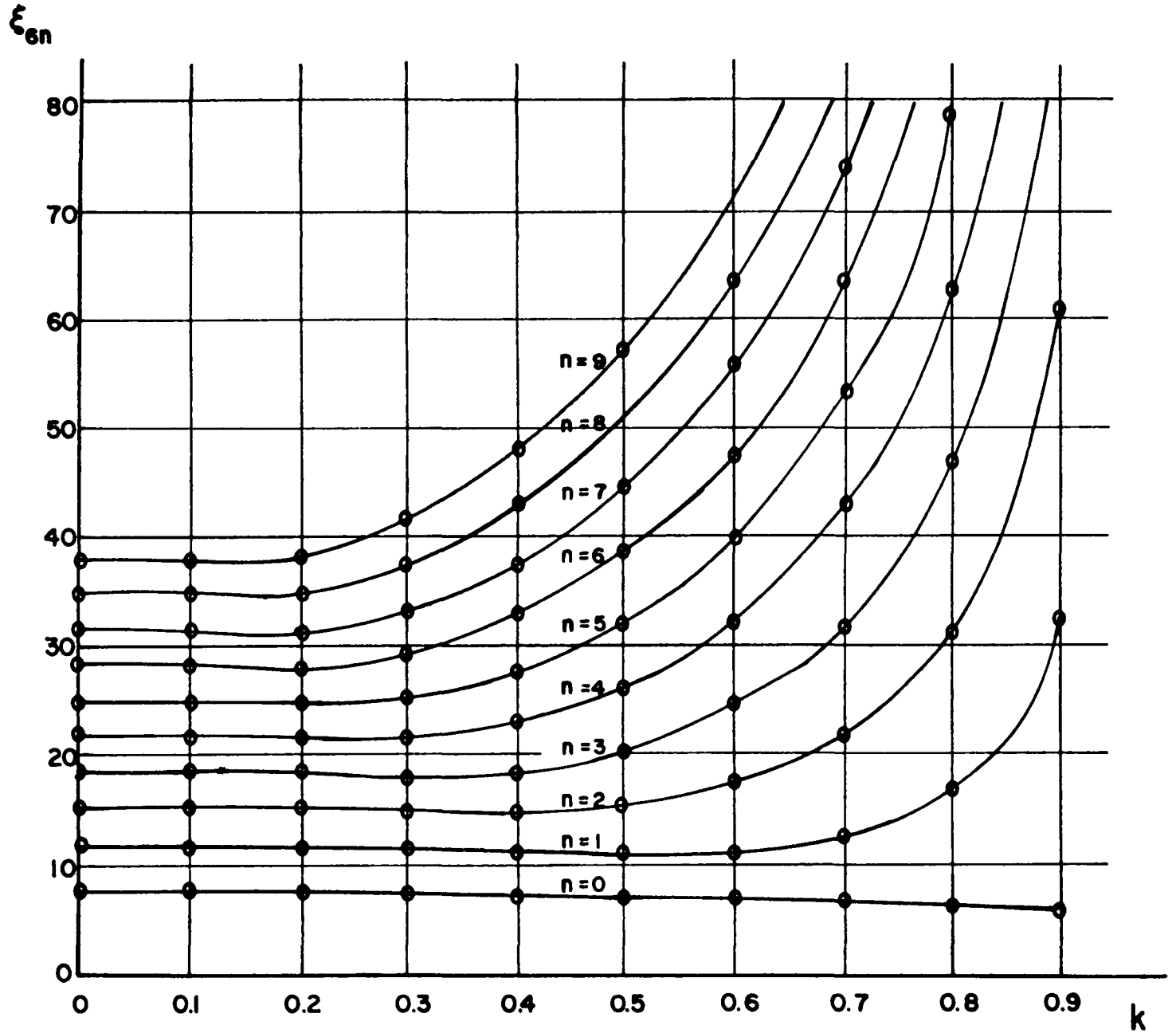

FIGURE 7. ROOTS $\Delta_6(\xi) = 0$ VS PARAMETER k

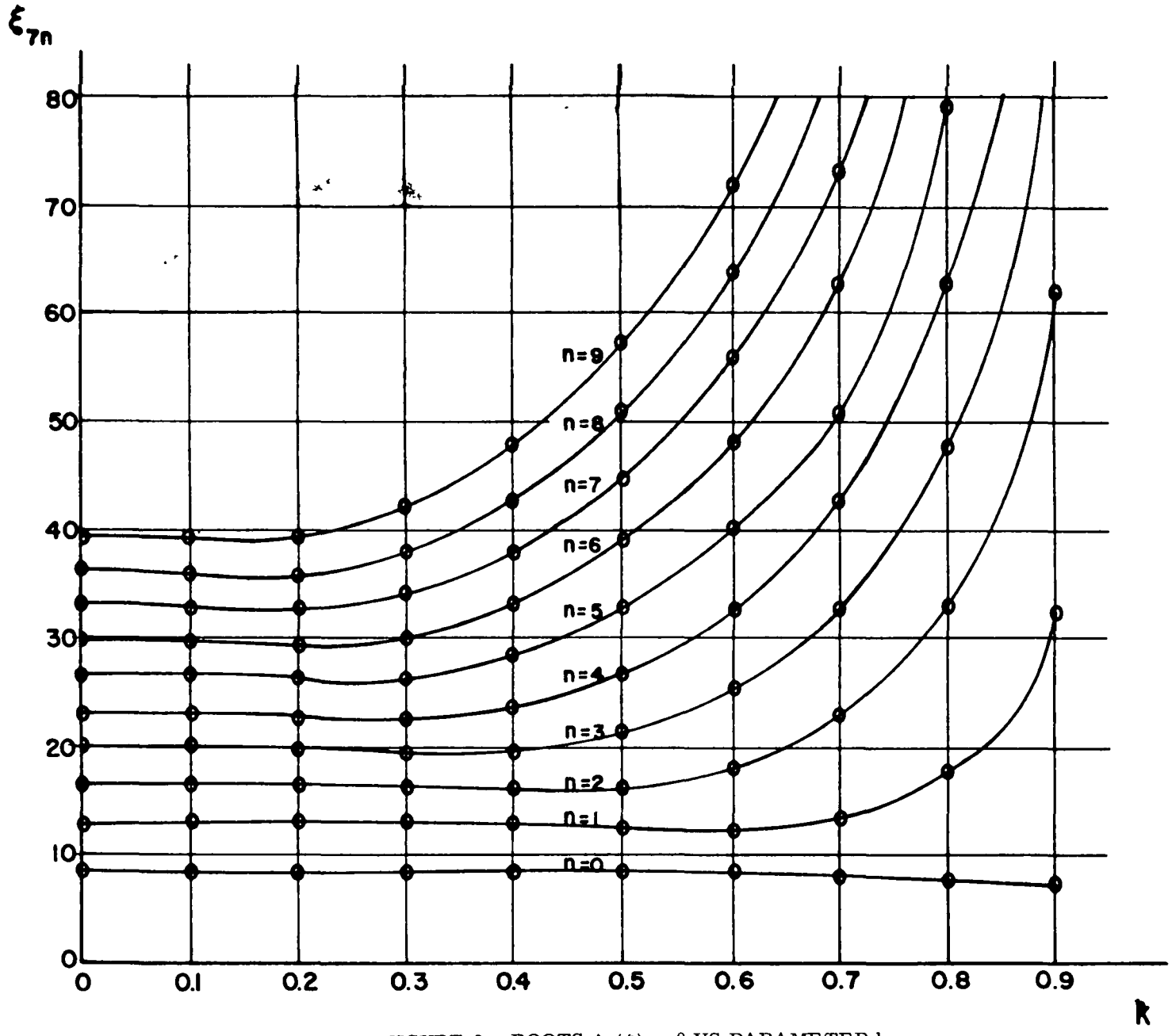

FIGURE 8. ROOTS $\Delta_7(\xi) = 0$ VS PARAMETER k

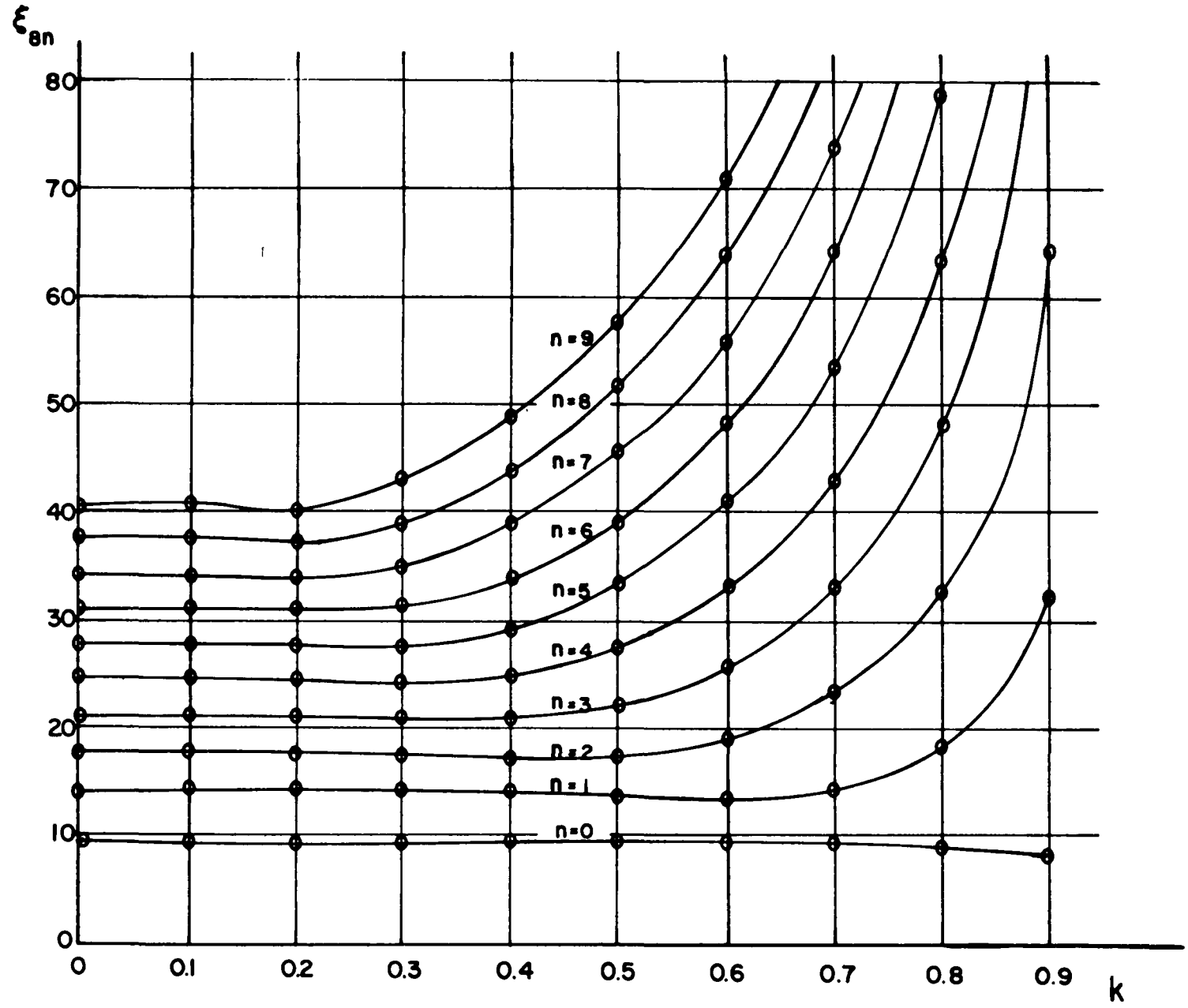

FIGURE 9. ROOTS $\Delta_8(\xi) = 0$ VS PARAMETER k

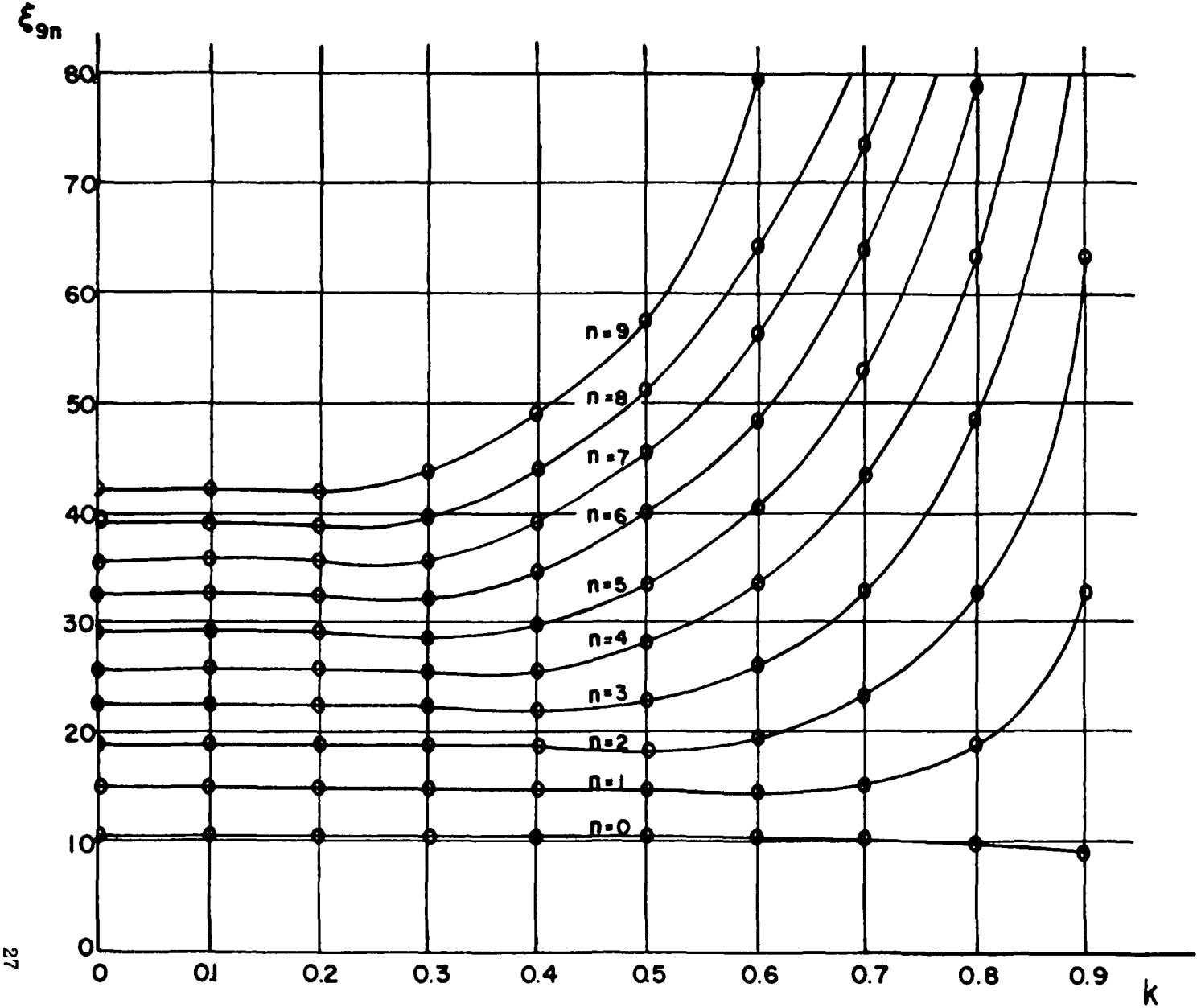

FIGURE 10. ROOTS $\Delta_9(\xi) = 0$ VS PARAMETER k

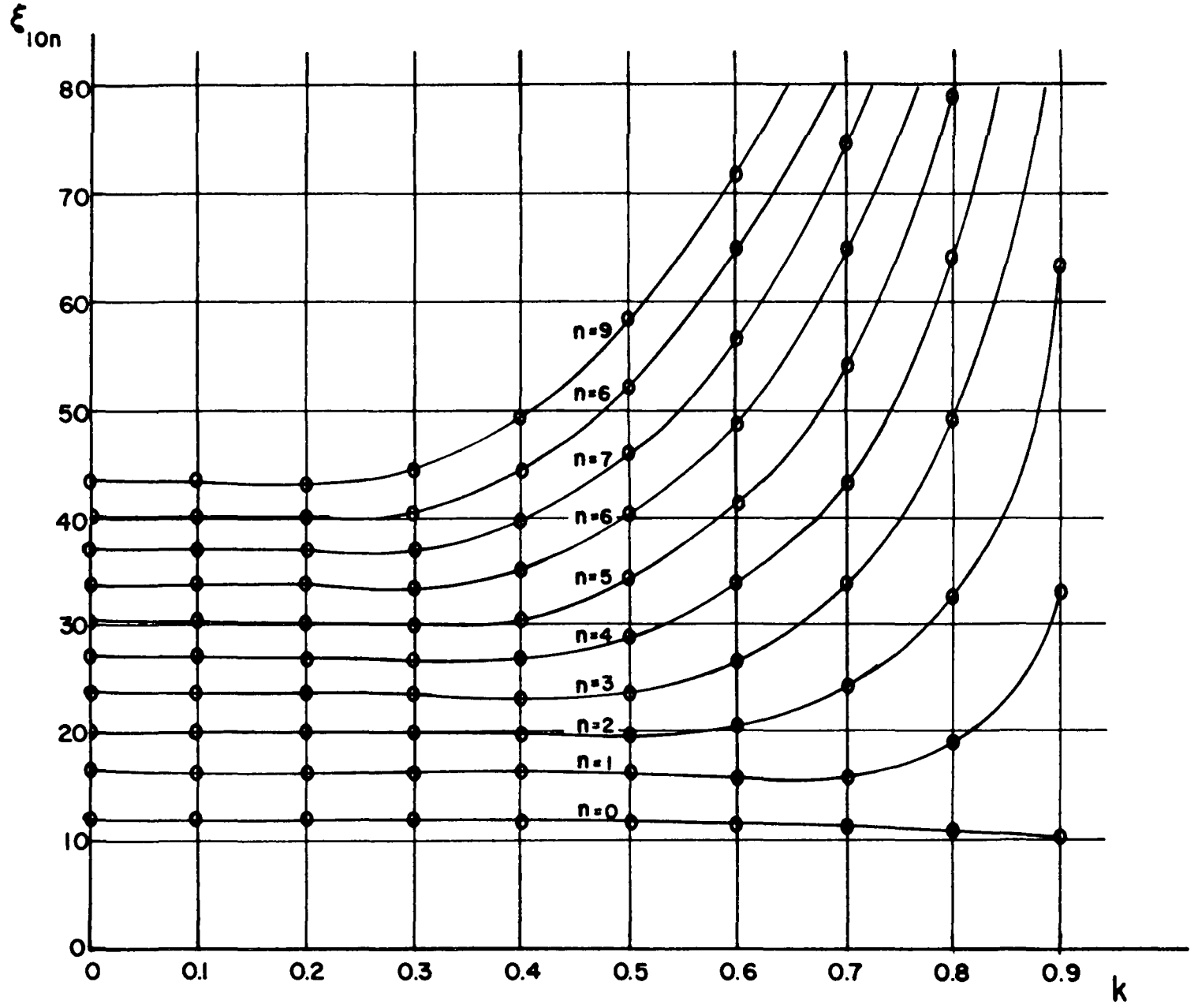

FIGURE 11. ROOTS $\Delta_{10}(\xi) = 0$ VS PARAMETER k

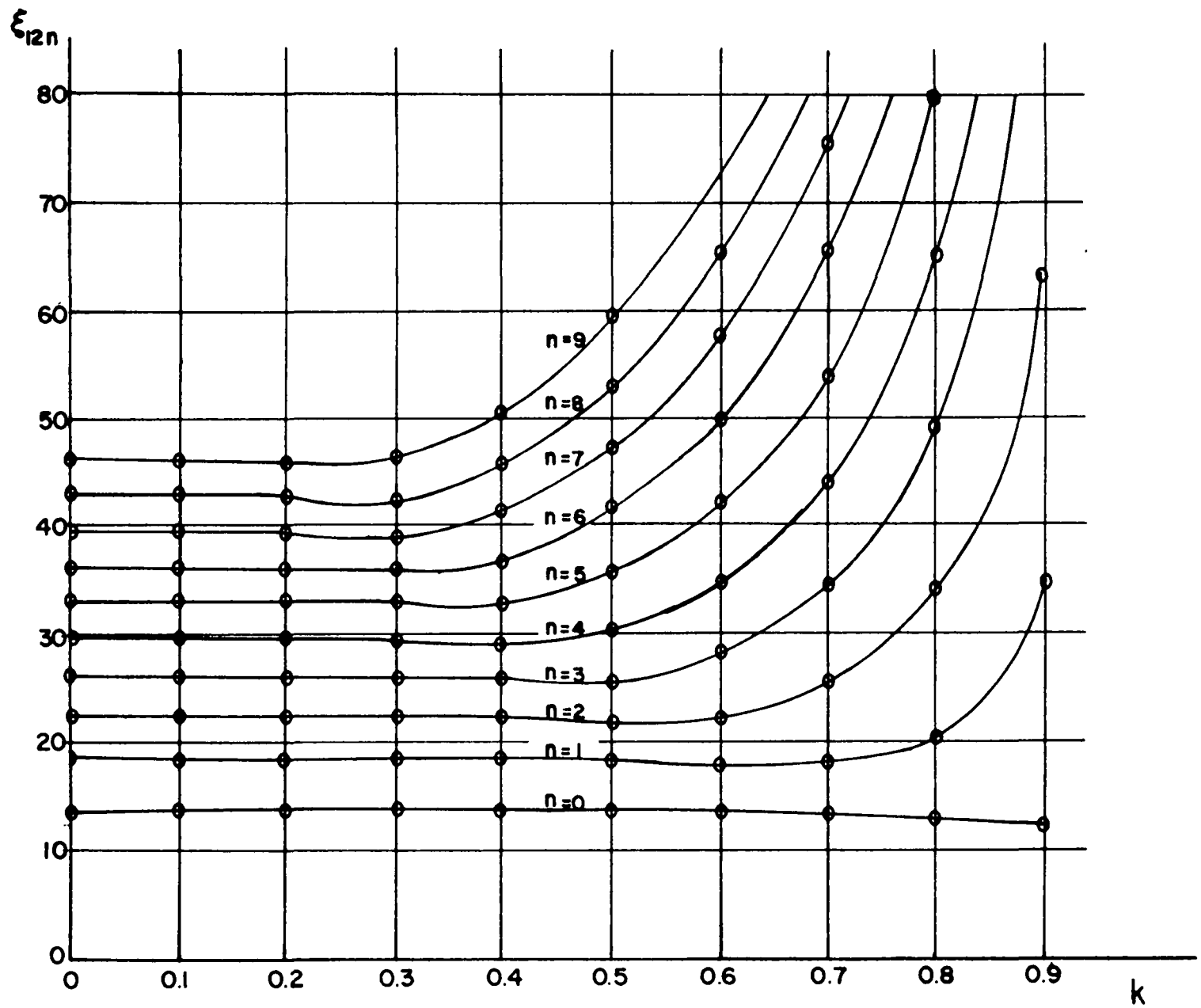

FIGURE 12. ROOTS $\Delta_{11}(\xi) = 0$ VS PARAMETER k

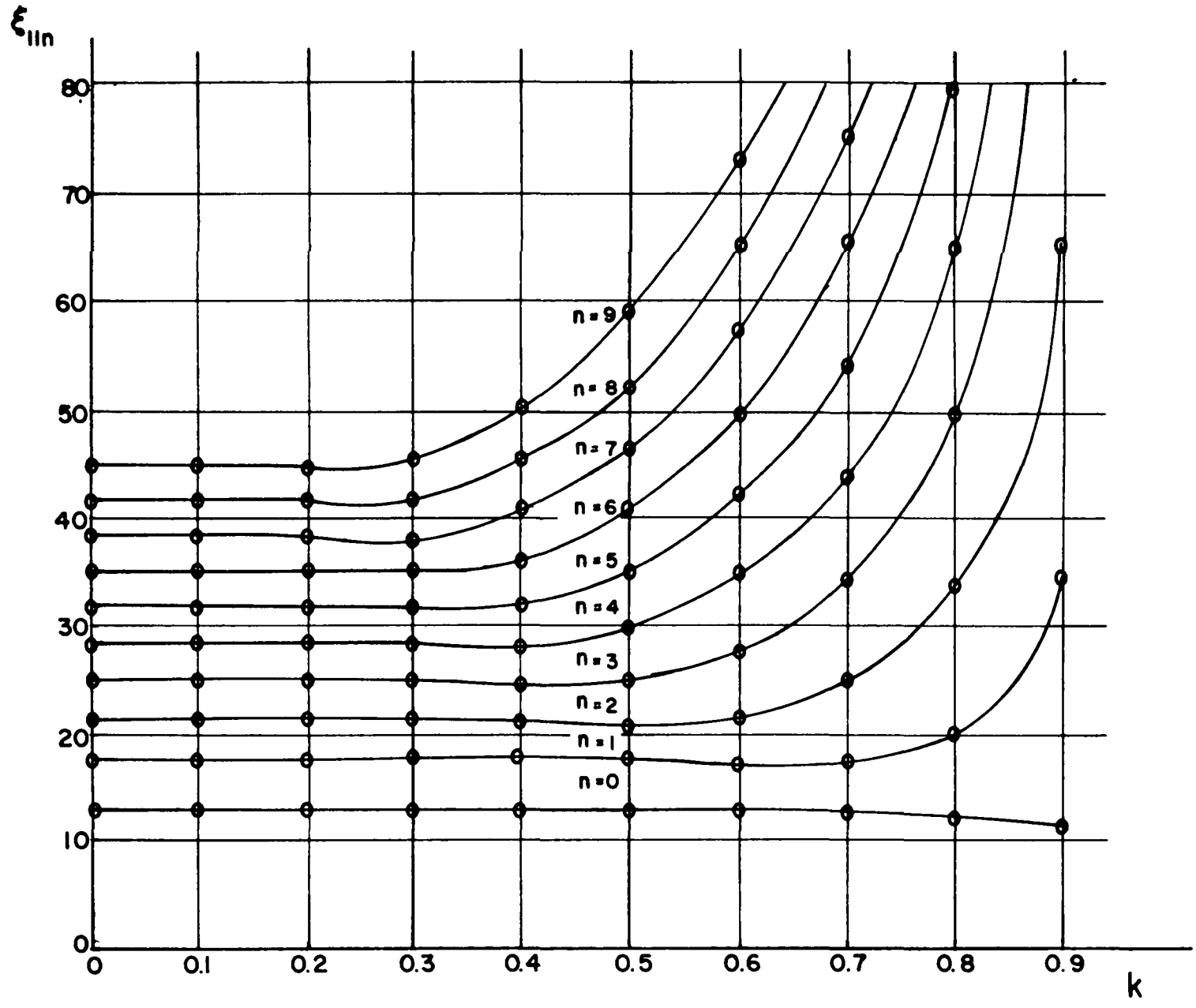

FIGURE 13. ROOTS $\Delta_{12}(\xi) = 0$ VS PARAMETER k

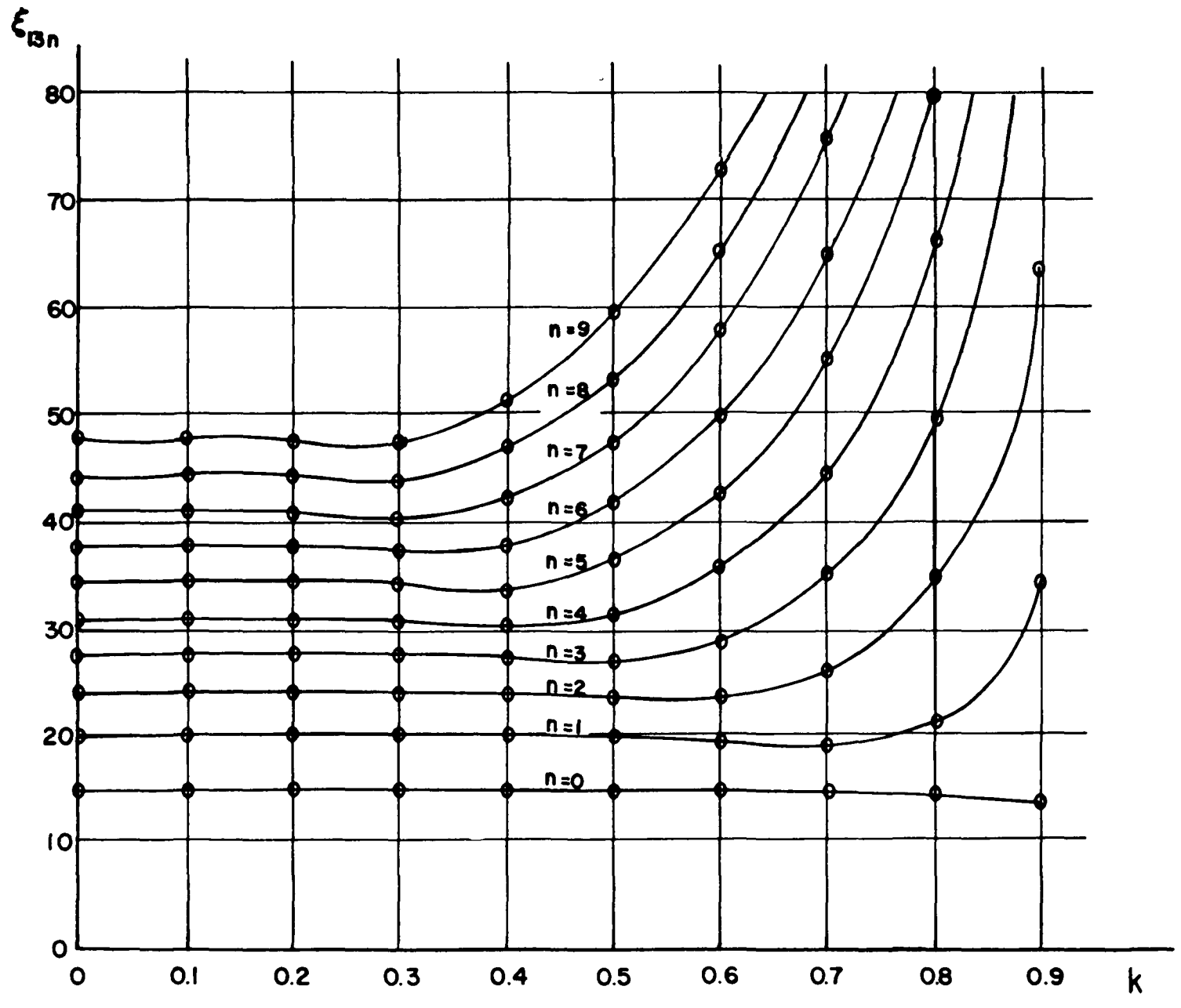

FIGURE 14. ROOTS $\Delta_{13}(\xi) = 0$ VS PARAMETER k

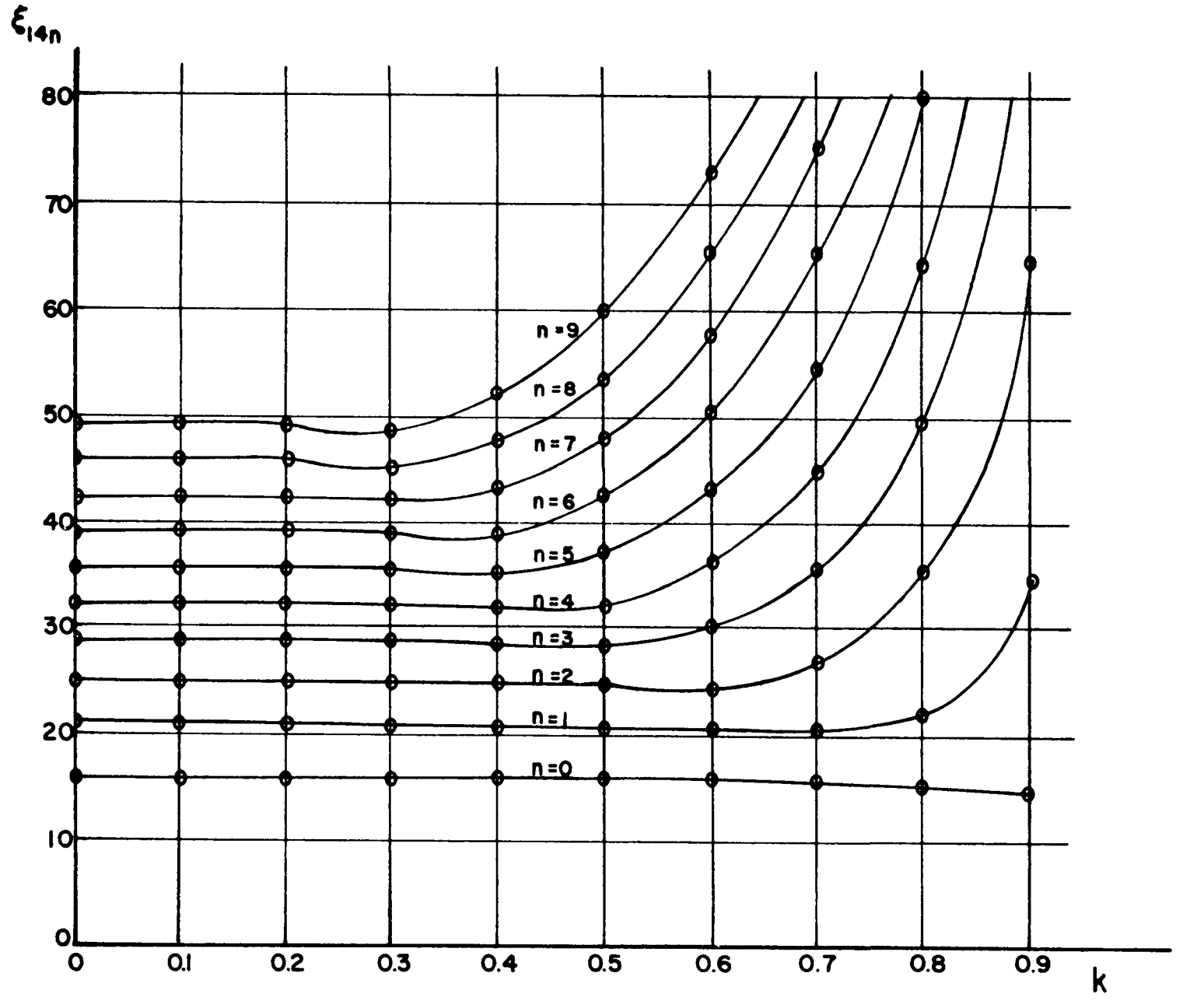

FIGURE 15. ROOTS $\Delta_{14}(\xi) = 0$ VS PARAMETER k

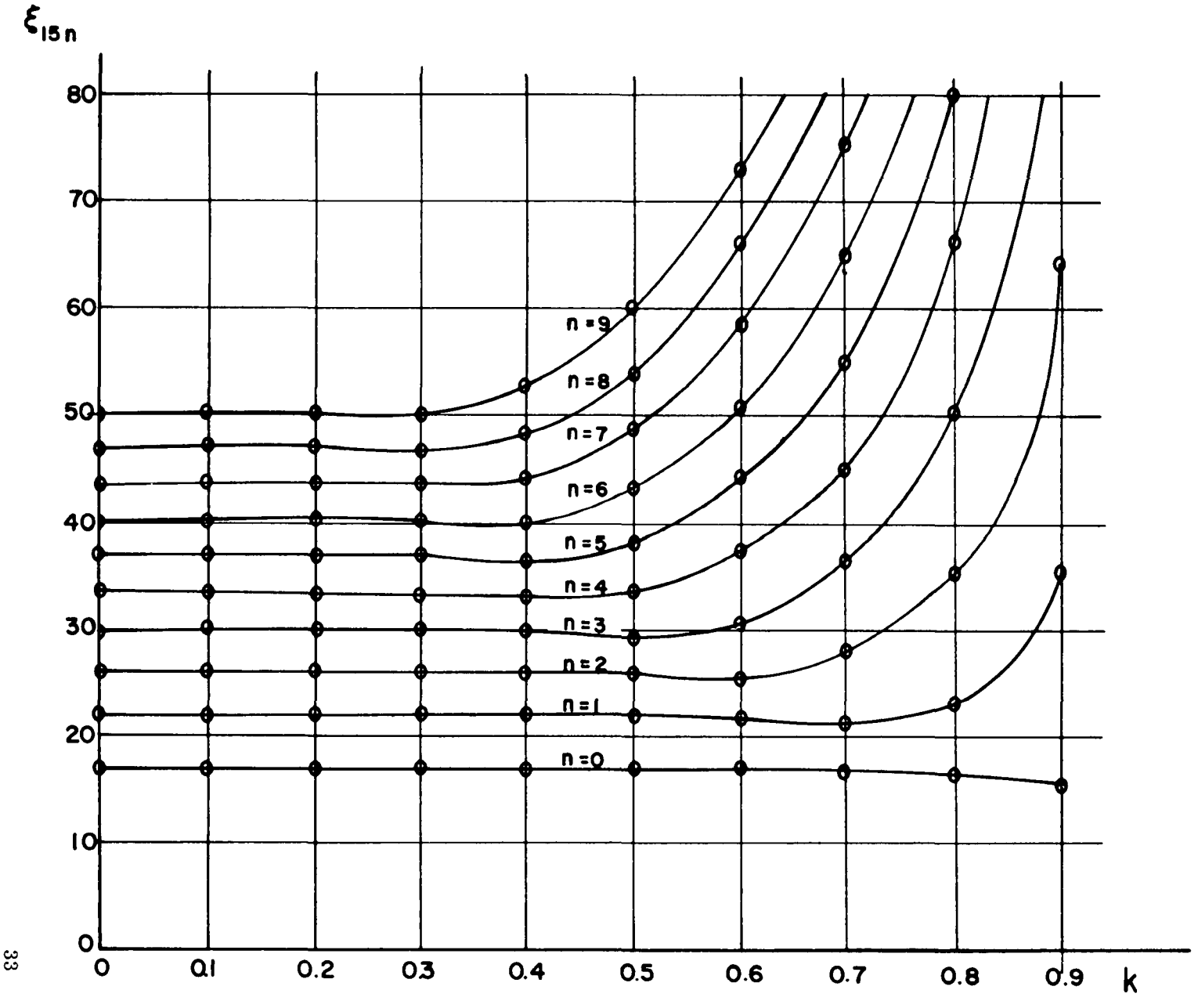

FIGURE 16. ROOTS $\Delta_{15}(\xi) = 0$ VS PARAMETER k

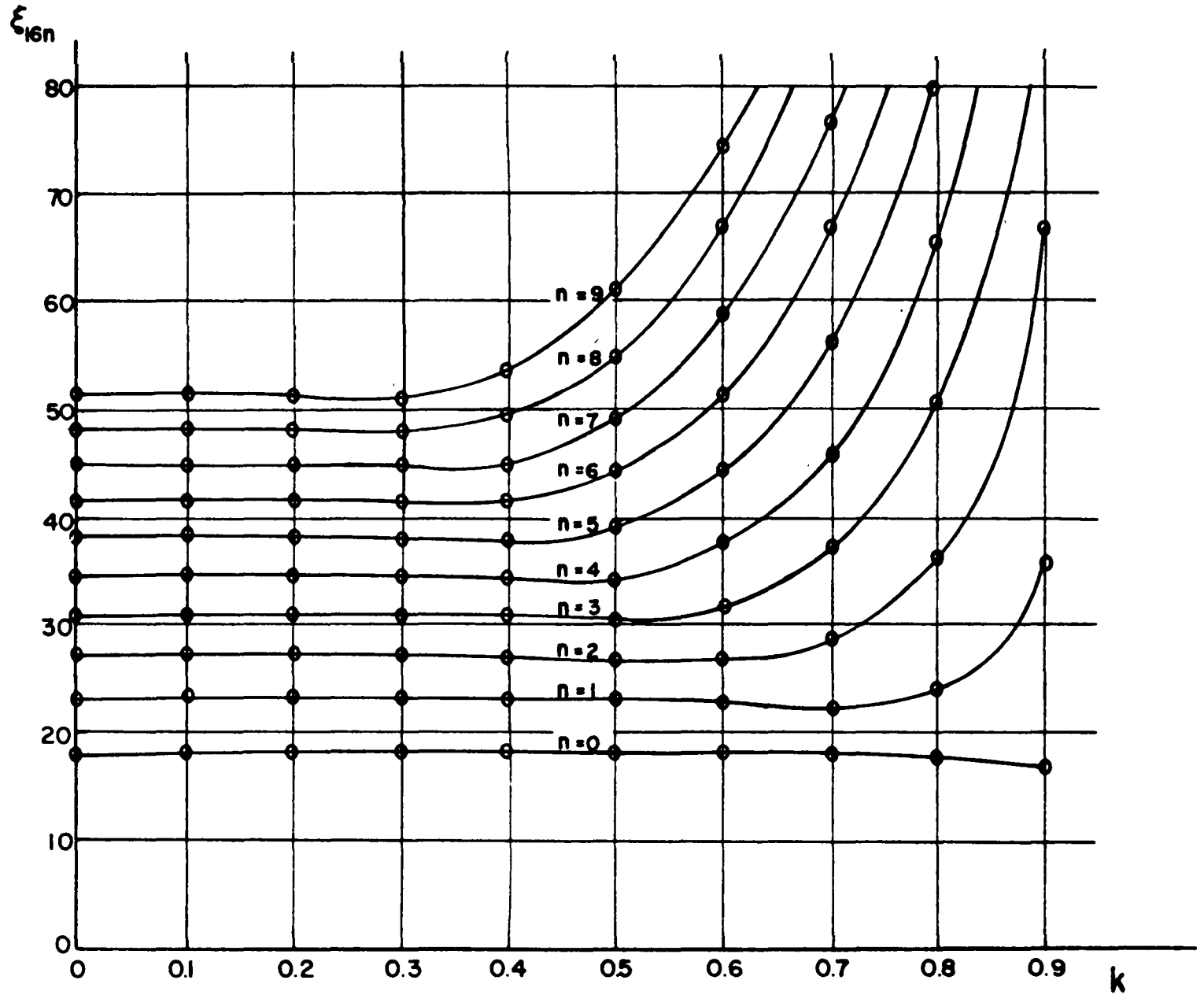

FIGURE 17. ROOTS $\Delta_{16}(\xi) = 0$ VS PARAMETER k

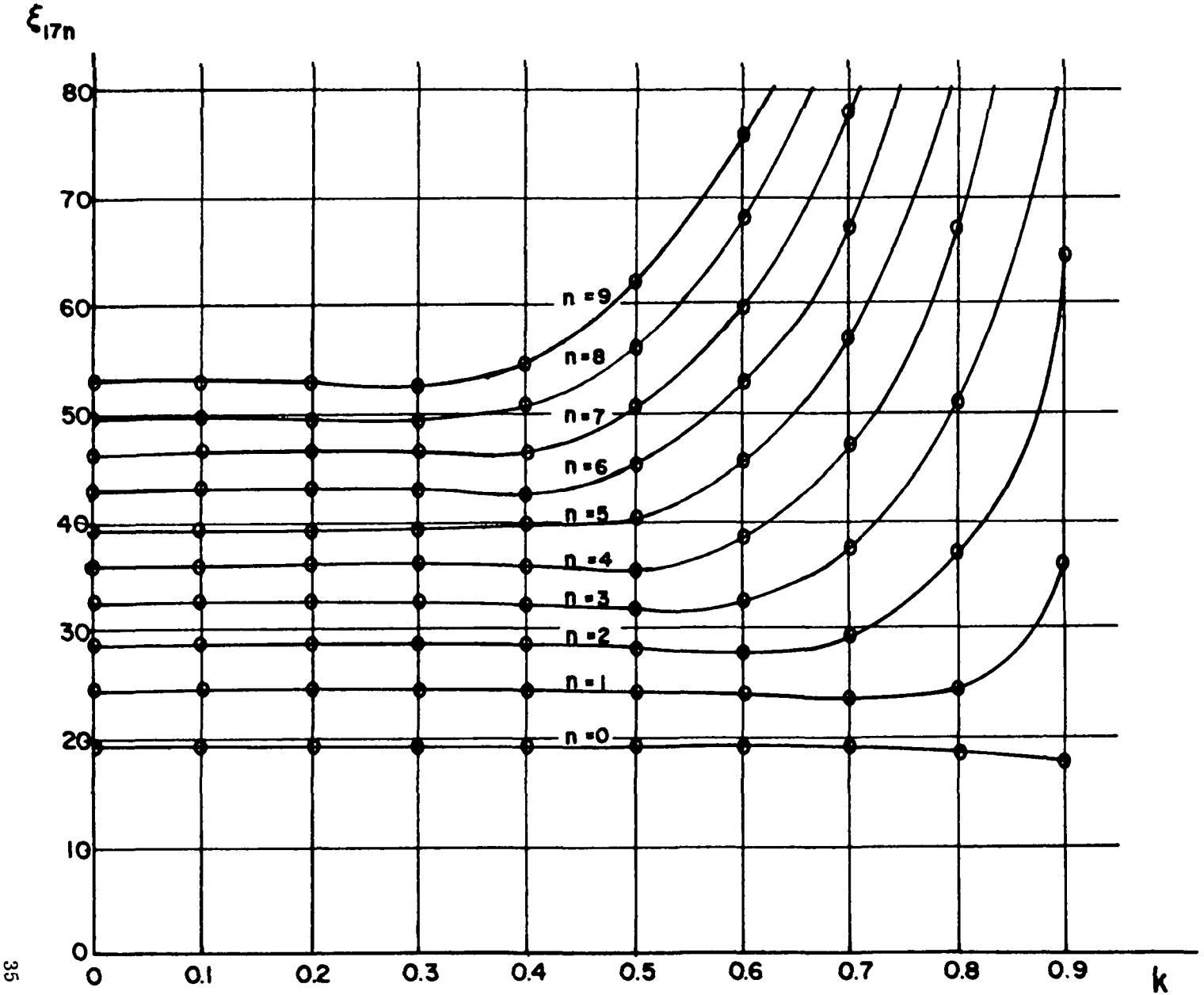

FIGURE 18. ROOTS $\Delta_{17}(\xi) = 0$ VS PARAMETER k

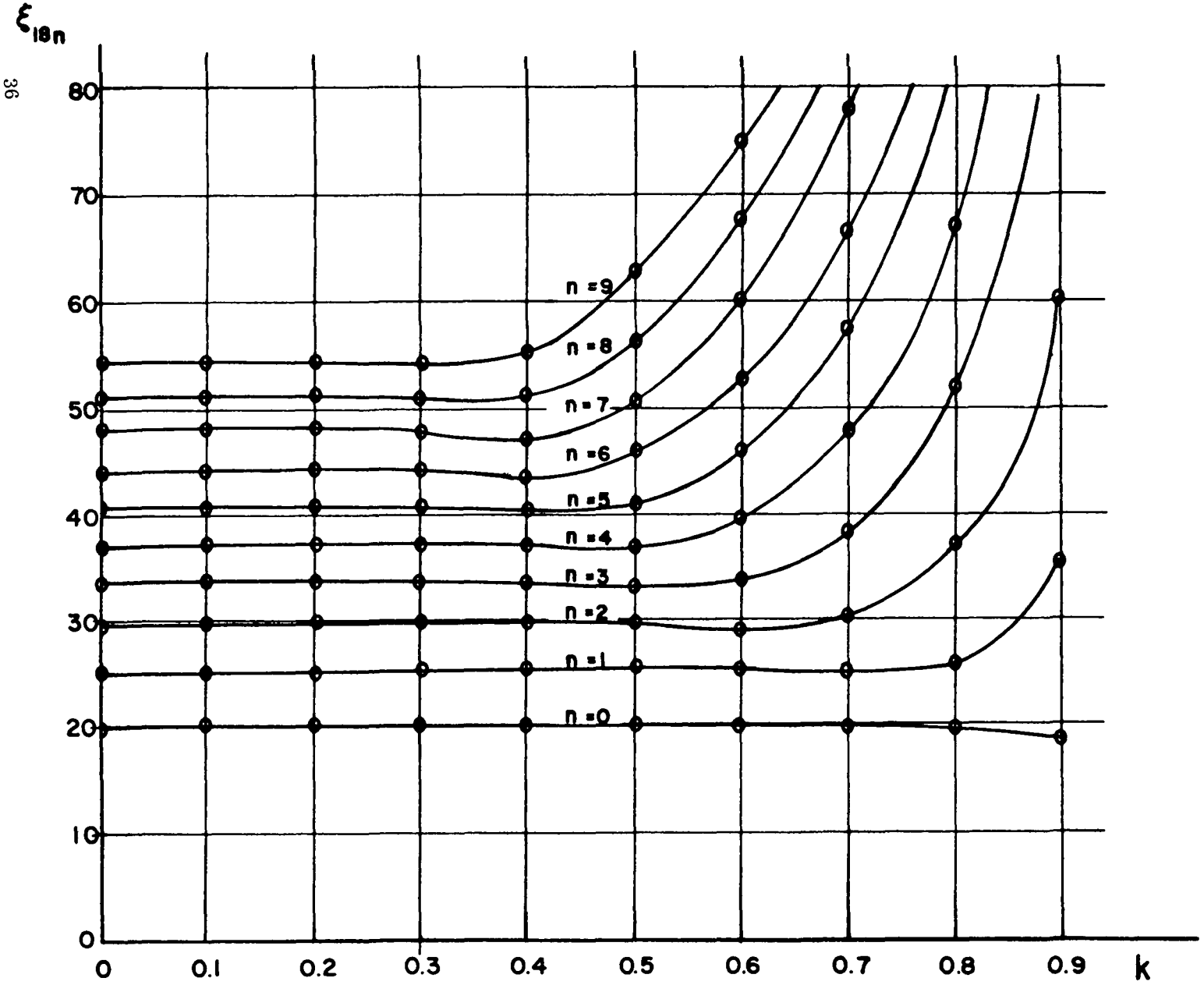

FIGURE 19. ROOTS $\Delta_{18}(\xi) = 0$ VS PARAMETER k

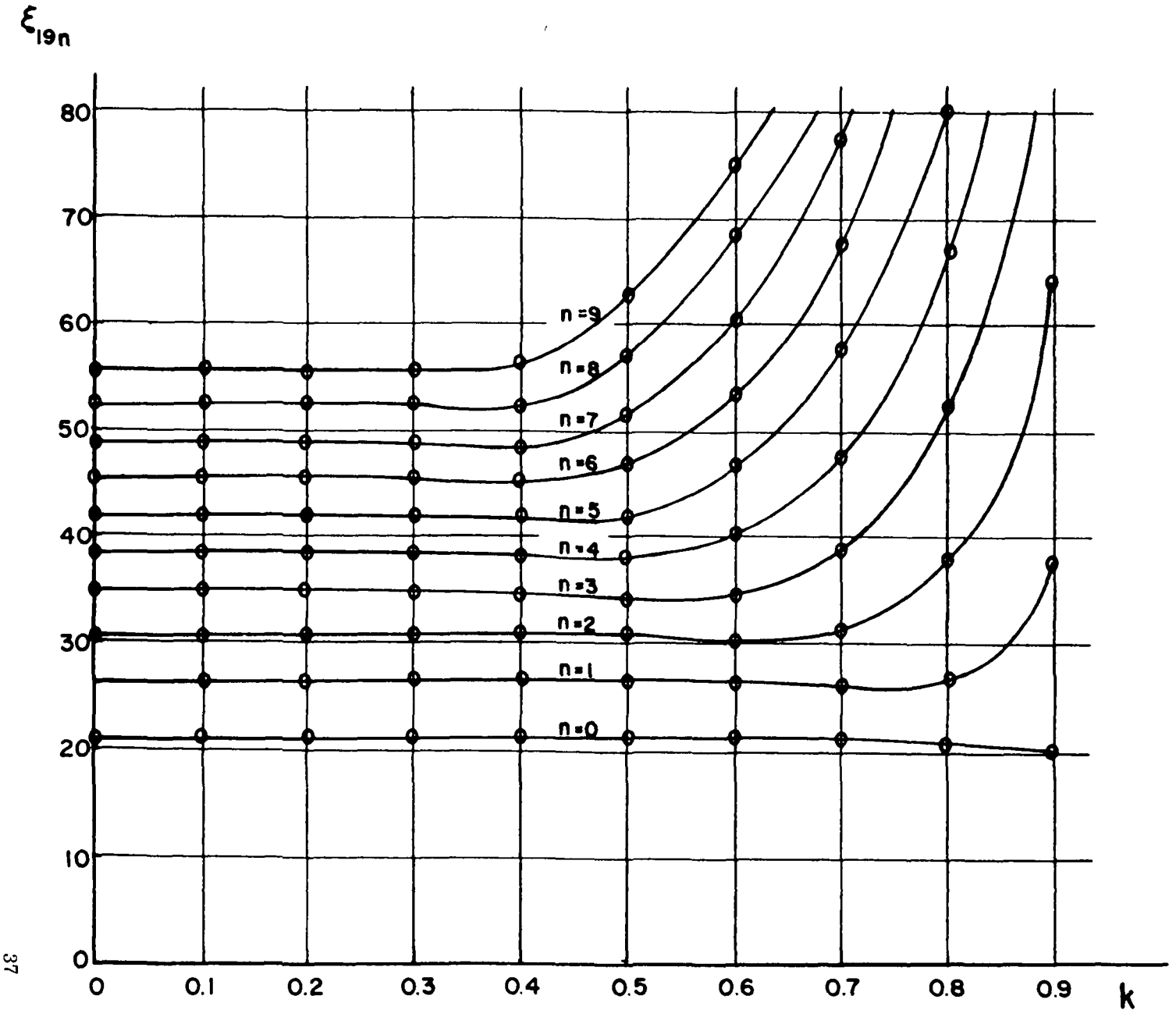

FIGURE 20. ROOTS $\Delta_{19}(\xi) = 0$ VS PARAMETER k

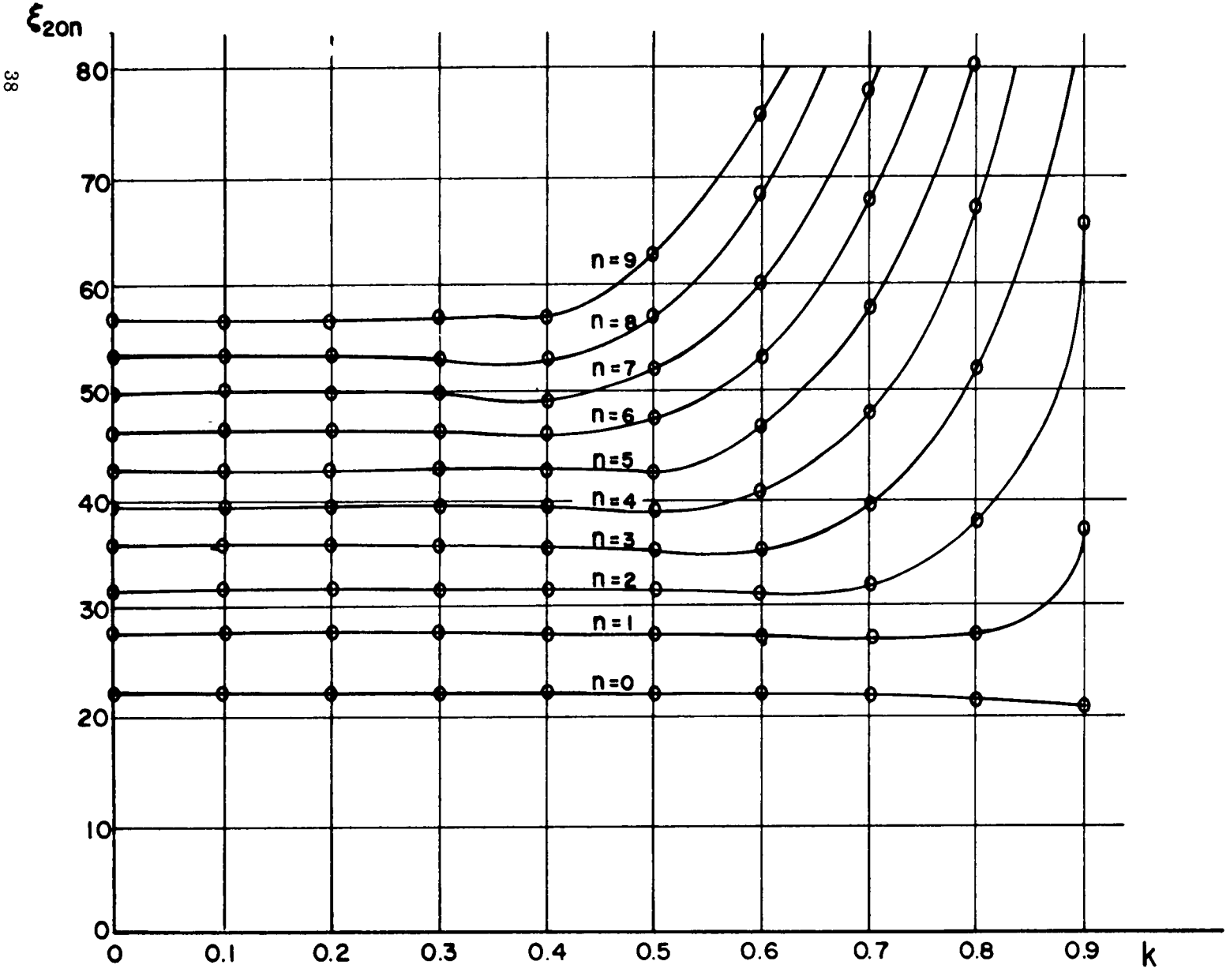

FIGURE 21. ROOTS $\Delta_{20}(\xi) = 0$ VS PARAMETER k

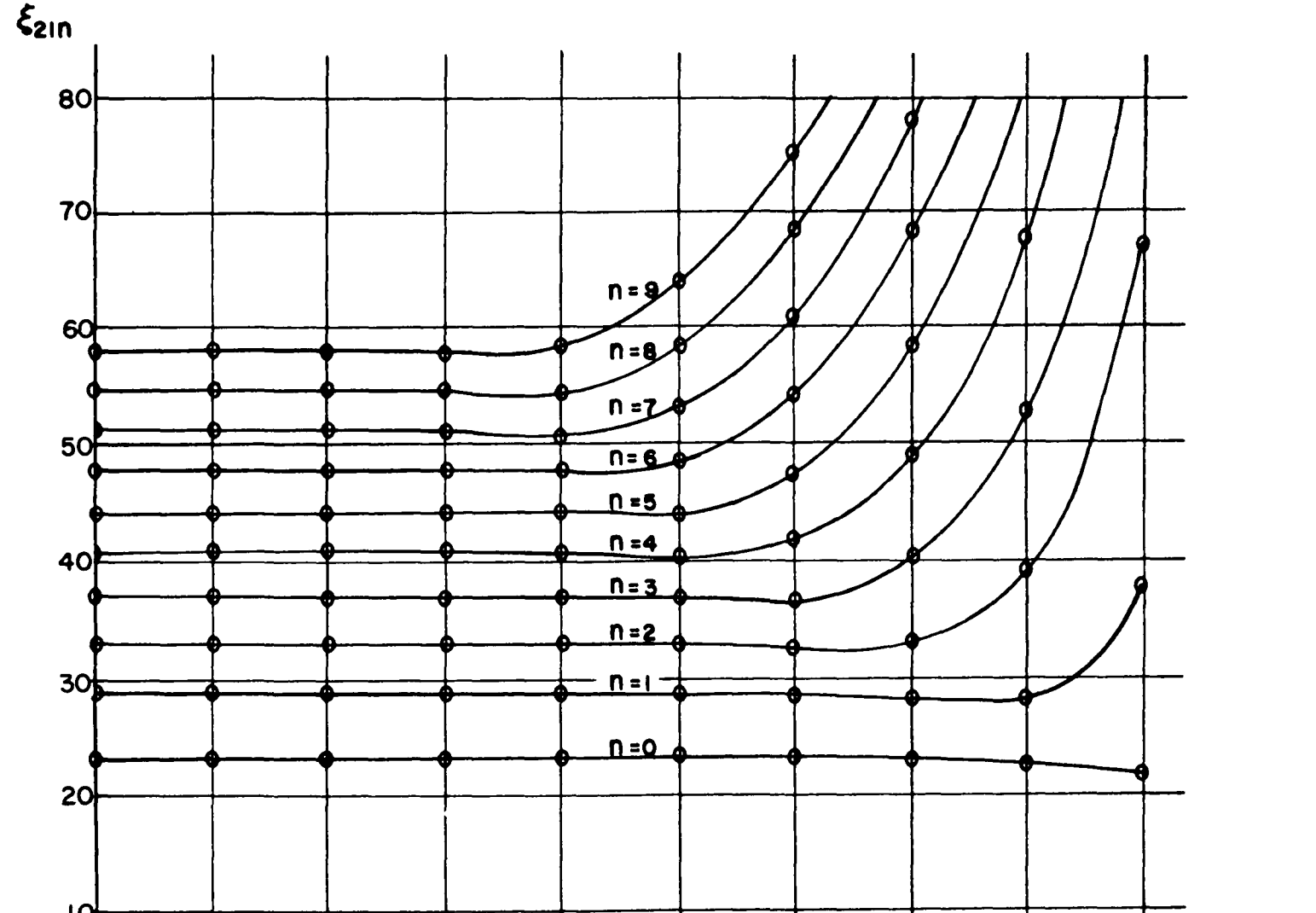

FIGURE 22. ROOTS $\Delta_{21}(\xi) = 0$ VS PARAMETER k

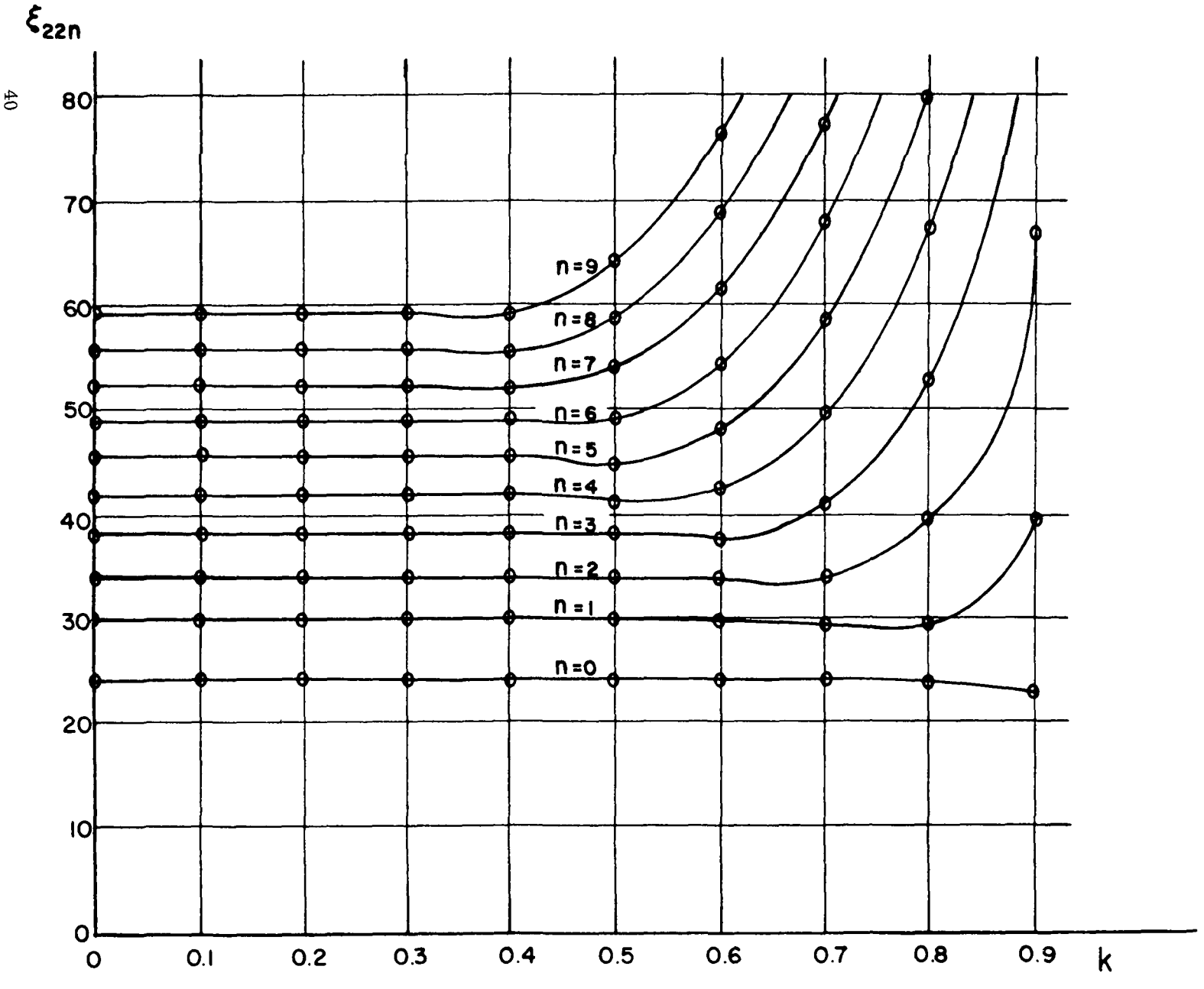

FIGURE 23. ROOTS $\Delta_{22}(\xi) = 0$ VS PARAMETER k

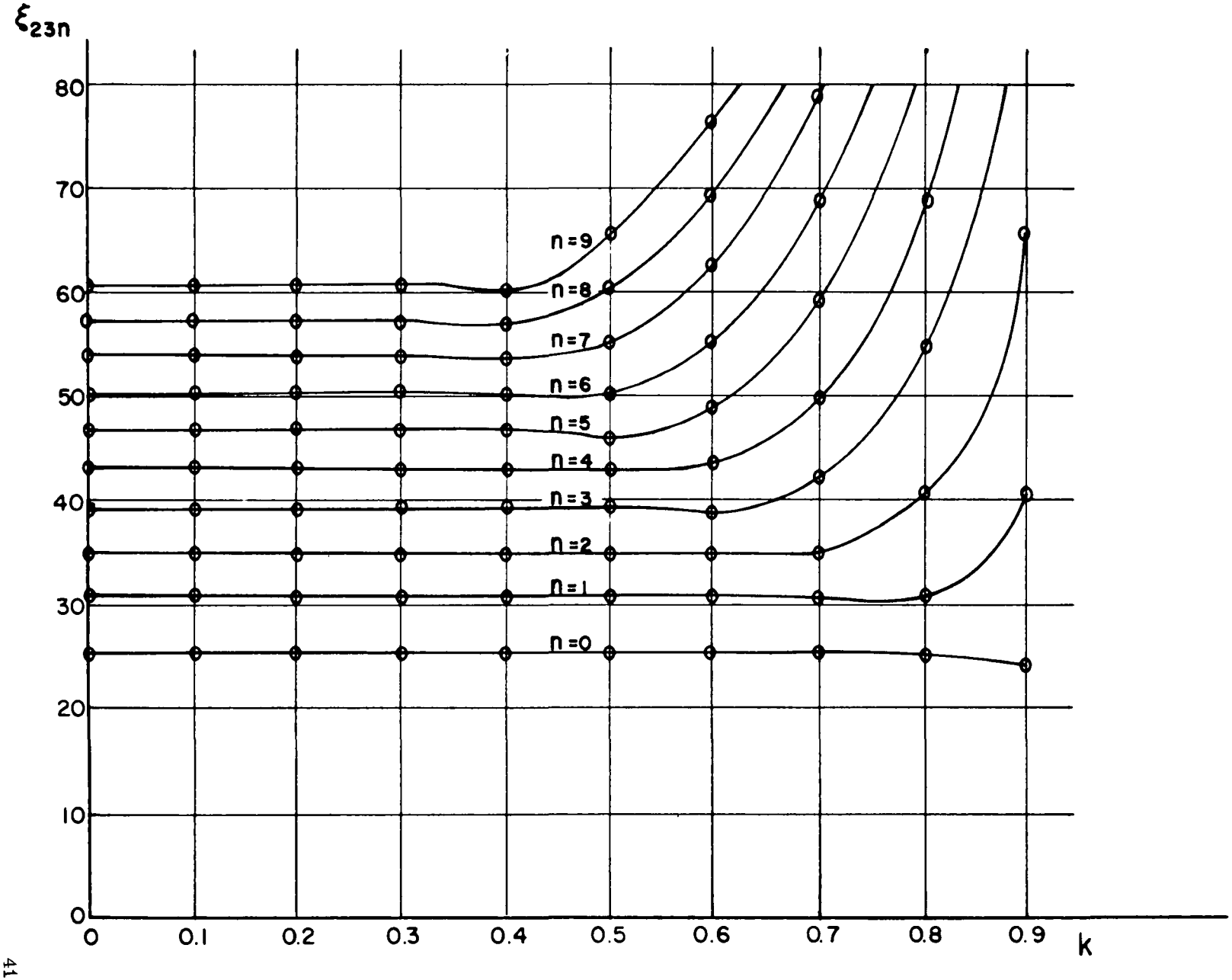

FIGURE 24. ROOTS $\Delta_{23}(\xi) = 0$ VS PARAMETER k

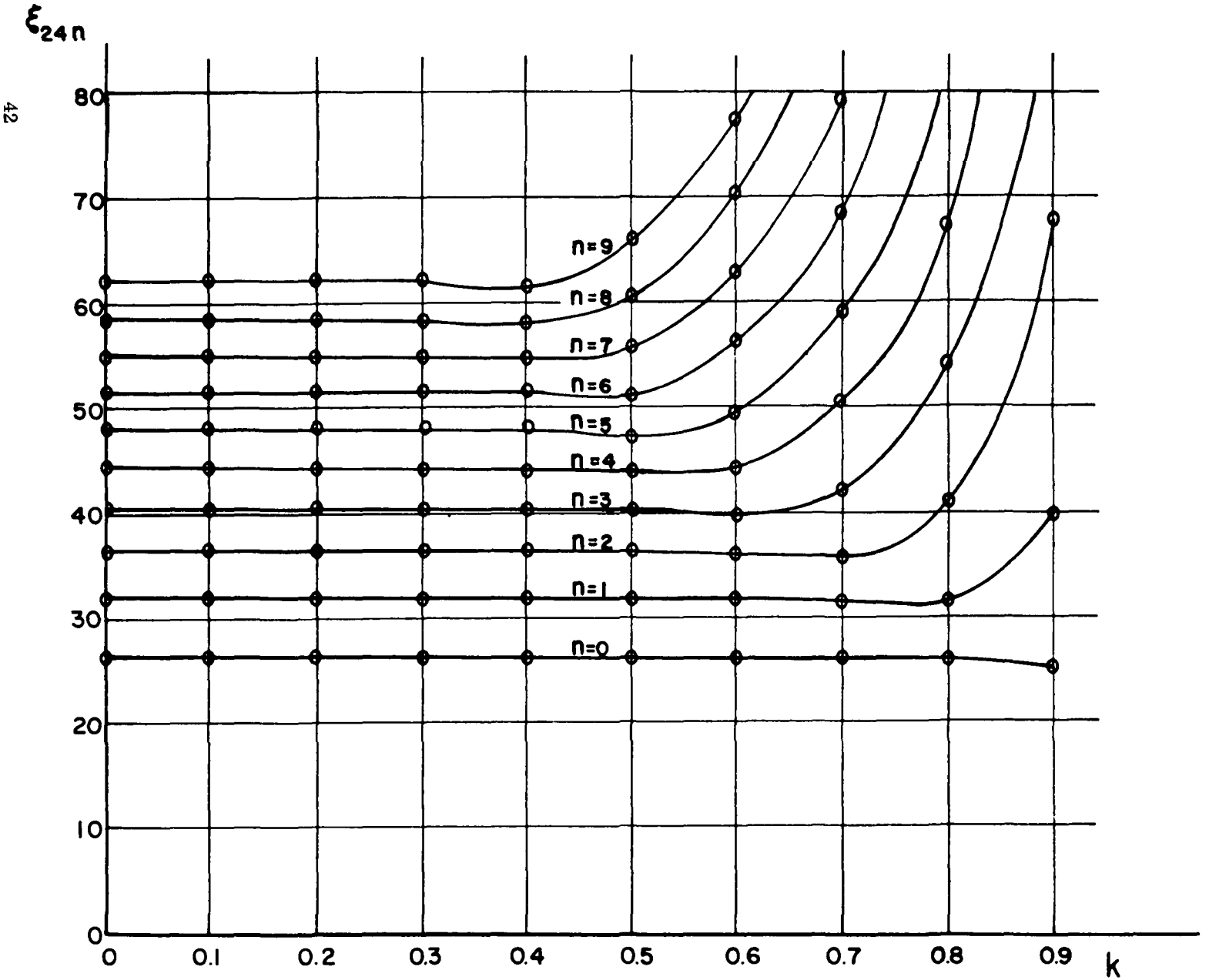

FIGURE 25. ROOTS $\Delta_{24}(\xi) = 0$ VS PARAMETER k

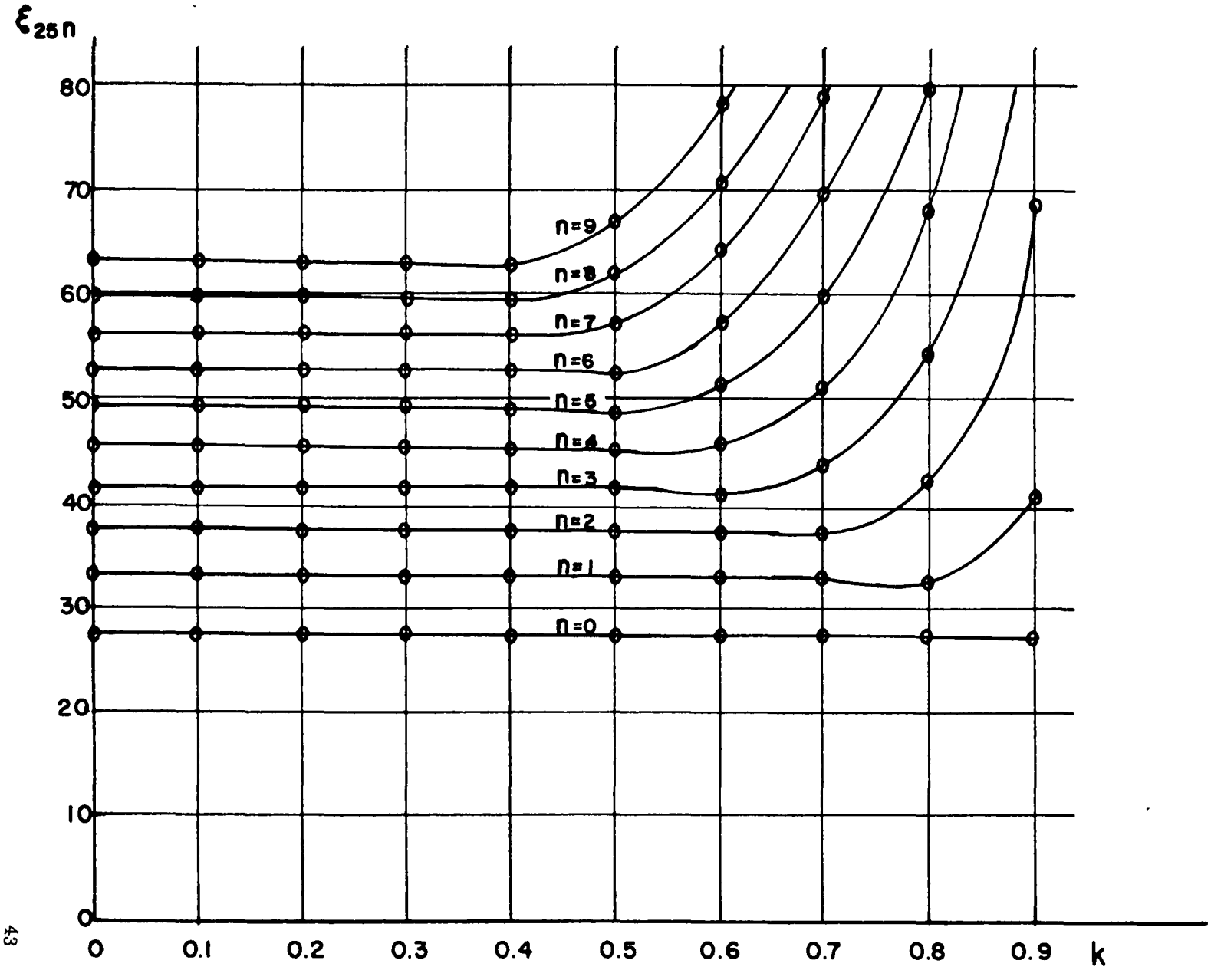

FIGURE 26. ROOTS $\Delta_{25}(\xi) = 0$ VS PARAMETER k

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TABLES AND GRAPHS OF ZEROS OF CROSS PRODUCT BESSEL FUNCTIONS
$J'_p(\xi)\ Y'_p(k\xi) - J'_p(k\xi)\ Y'_p(\xi) = 0$

By

Helmut F. Bauer

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